Gradient Vector Flow Fast Geometric Active Contours

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Abstract—In this paper, we propose an edge-driven bidirectional geometric flow for boundary extraction. To this end, we combine the geodesic active contour flow [3] and the gradient vector flow external force for snakes [25]. The resulting motion equation is considered within a level set formulation [19], can deal with topological changes and important shape deformations. An efficient numerical schema is used for the flow implementation that exhibits robust behavior and has fast convergence rate [8], [23]. Promising results on real and synthetic images demonstrate the potentials of the flow.

Index Terms—Boundary extraction, image segmentation, gradient vector flow, geodesic active contours, level set methods, additive operator splitting.

1 Introduction

Boundary extraction and image segmentation are quite popular low-level topics of research in image processing and computer vision. These problems have been dealt with in various forms. Variational approaches is an emerging formulation to tackle such form of applications where the objective is to create an image partition that follows the data while preserving some regularity constraints. To this end, deformation of geometric structures as well as propagation of curves under the influence of internal and external forces have been considered.

Such propagation can either be the outcome of an optimization process [3], [9] or defined in the form of a geometric flow [2], [16], [25]. An attempt to classify these techniques can lead to the following distinct categories:

Deformable Templates: This class of approaches is the least flexible one since it relies on models that are made up of low-level geometric components such as lines, circles, and ellipses [15].

Point Distribution Models: A step further to the rigid and articulated models was the use of specific patterns of variability from a representative training set [5]. To this end, the model parameters are derived from the distribution of the training set of examples.

Snakes and Active Contour Models: The snake model [9] was a major breakthrough in computer vision and made the use of curve propagation approaches quite efficient and popular. It refers to an elastic curve, locally deformed by internal and external forces toward the lowest potential of an image-based cost function.

Geometric Flows: A novel class of techniques based on the propagation of planar curves [2], [16], [26] where image-driven forces are combined with internal terms to derive propagation flows able to deal with various applications (i.e., boundary extraction, segmentation, stereo, tracking, etc.).

The initial conditions and the ability to change the topology of the evolving curve are the most important limitations for most of these methods. Constraints on the initial conditions are imposed by the use of gradient terms to reach object boundaries. Such terms quite often produce one-directional boundary-driven propagation flows. Regional forces that aim at separating the intensities between the objects and the background have been used mainly to deal with this limitation [28]. Such components can be very efficient but require prior knowledge on the object intensity properties. On the other hand, recently, some boundary-based techniques were proposed to address this limitation [13], [21], [22]. Among these techniques, a quite promising solution was introduced in [25], [24], [26].

The use of Lagrangian approaches to implement such flows imposes the constraint of a static topology. Knowledge-based segmentation quite often does not require the ability of changing the topology of the evolving curve. However, for some cases, the structure of interest can change the topology. In order to deal with topological changes, some variations of the Lagrangian implementation were proposed in [6], [17]. A natural formulation to address this limitation was derived in [3], [11].

The geodesic active contour can be considered as a geometric alternative to the snake model for boundary extraction. This model was efficiently combined with techniques from fluid dynamics (the level set method [19]), leading to a paradigm that can deal with important shape deformations and topological changes. In [2], [16], one can find precedent geometric flows with behavior similar to the geodesic active contour.

In this paper, we propose an edge-driven bidirectional geometric flow for boundary extraction [21]. Our approach integrates three different aspects: 1) the gradient vector flow [25] to determine the image-based propagation force, 2) the geodesic active contour flow [3] to determine the geometric flow, and 3) the level set technique [19] to implement the resulting flow using a fast/robust numerical method [23], [8]. The most closely related work with our approach can be found in [3], [12], [13], [22], [25], [26]. Our flow can be considered as a subcase of the family of flows presented independently in [26] for boundary extraction. However, the concept is derived using a different motivation path.

The remainder of this paper is organized as follows: In Section 2, we present some concepts on propagation of curves, while, in Section 3, the gradient vector flow estimation as well as its elaboration to the geodesic active contour motion equation are considered. Implementation details and experimental results are presented in Section 4. Discussion appears in Section 5.

2 On the Propagation of Curves

The snake model [9] matches a curve to an image by means of cost function optimization. It exhibits dynamic behavior and has been used to deal with a wide variety of applications. In the most general case, the objective function consists of an **internal** term that stands for the curve regularity and an **image** term that is derived from the observed data and attracts the curve to the desired features. Last, but not least, quite often an **external** term derived by some user-defined constraints is considered to decrease the dimensionality of the solution space.

Such a model is a powerful interactive tool to deal with frame partition problems in computer vision. However, one can claim it is "myopic" since it only makes use of local information and is sensitive to the initial conditions. Another limitation of the method is associated with the fact that the model depends on the parameterization of the curve that is not natural for a boundary extraction approach. To cope with these issues, numerous variations of the original snake model can be found in the literature.

2.1 Geodesic Active Contour

The geodesic active contour model [3], [11] was introduced as a geometric alternative for snakes. The objective function associated with this framework is the following:

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$$E[(C)(p)] = \underbrace{\int_0^L g(|\nabla I(C(s))|)ds}_{Geodesic\ Active\ Contour} = \int_0^1 \underbrace{g(|\nabla I(C(p))|)}_{attraction\ term} \underbrace{\left|\frac{\partial C}{\partial p}(p)\right|}_{regularity\ term} dp, \quad (1)$$

where g a positive defined monotonically decreasing function with the following properties $g : [0, +\infty] \to \mathcal{R}^+$, $g(0) = 1, g(x) \to 0$ as $x \to \infty$, ds is the Euclidean arc-length element, and L the Euclidean length of $C(p)$. Within this framework, object detection is equivalent to finding the minimal length geodesic curve that best takes into account the desired image characteristics (important spatial image gradients).

One can recover the lowest potential of this function using a gradient descent method and the calculus of variations by evolving an initial curve $C_0(.)$ according to the following flow:

$$C_t(p) = \underbrace{g(|\nabla I(p)|)\mathcal{K}(p)\mathcal{N}(p)}_{data\ attraction\ and\ smoothness} - \underbrace{(\nabla g(|\nabla I(p)|) \cdot \mathcal{N}(p))\mathcal{N}(p)}_{bi\text{-}directional\ boundary\ force}, \quad (2)$$

where $\mathcal{N}$ is the inward Euclidean normal and $\mathcal{K}$ the curvature. According to this flow, each contour point moves along the normal toward the direction that decreases the weighted length of C. There are two forces acting on the contour, both in the direction of the normal: 1) a force that mainly shrinks/expands (modulo the curvature effect) the curve toward the object boundaries and 2) a bidirectional force applicable in the vicinity of the object boundaries that can deal with concave regions.

2.2 The Level Set Method

The implementation of this evolution can be done using a Lagrangian approach. In the most general case, the evolving interface cannot change topology. To overcome these limitations, Osher and Sethian [19] have proposed representing the evolving interface $C(p)$ with a zero-level set ($\Phi(C(p) = 0)$ function: $\left[(x, y, \Phi(x, y, t)) \in \mathcal{R}^3\right]$. Let us consider the follow motion equation

$$C_t(p) = F(\mathcal{K}(p))\ \mathcal{N}(p) \quad (3)$$

and F is a continuous function of the curvature $\mathcal{K}$. Then, deriving $\Phi(C(p), t) = 0$ with respect to time and space (given (3)) we obtain the following motion for the embedding surface $\Phi()$:

$$\frac{d}{dt}\Phi(x, y) = F(\mathcal{K}(x, y))\ |\nabla\Phi(x, y)|, \quad (4)$$

where $[|\nabla\Phi|]$ is the norm of gradient and $[\mathcal{N} = -\frac{\nabla\Phi}{|\nabla\Phi|}]$. Thus, we have established a connection between the family $C(p, t)$ and the one parameter surfaces $\Phi(x, y, t)$. Such a connection is through the zero iso-surface of the function Φ that always yields to the evolving interface. The level set formulation is implicit, intrinsic, parameter-free, and can change the topology of the evolving interface.

The geodesic active contour when implemented using the level set method does not depend on the curve parameterization and is relatively free of the initial conditions. However, it relies on a nonparameterized curve that evolves mainly in one direction (inward or outward) toward the objects despite the effect of the second bidirectional force with limited captivity. Thus, in practice, proper use requires a *specific* initialization step, where the initial curve should be completely exterior or interior to the real object boundaries.

Many efforts have been made to overcome these shortcomings by introducing some region-based features that make the model free of the initial conditions and more robust [28], [4], [20], [27]. Such techniques can have exceptional performance under certain conditions. The force derived from the data is single-directional and aims at shrinking the contour toward the object boundaries. The use of region-driven modules along with regularity constraints is the most efficient way to deal with this limitation. To this end, boundary information is not considered within the optimization process.

3 Gradient Vector Flow and Geometric Active Contours

The gradient vector flow (GVF) [25] refers to the definition of a bidirectional external force that can capture the object boundaries from either sides and can deal with concave regions. Such a field is recovered through the diffusion of the edge-driven information and has an interpretation similar to the optical flow. One can interpret this field as the direction to be followed to reach the object boundaries.

3.1 Construction of the Gradient Vector Flow

Toward the construction of this field, one first has to define an appropriate continuous edge detector. To this end, we consider a Gaussian-derived function on the image gradient [7]:

$$\left[f(I(p)) = 1 - \frac{1}{\sqrt{2\pi}\sigma_E} e^{-\frac{|\nabla(G_\sigma * I)(p)|^2}{2\sigma_E^2}}\right],$$

with σ_E variance, where $[G_\sigma * I]$ is the convolution output of the input image with a Gaussian Kernel (smoothing).

The gradient vector flow [25] consists of a two-dimensional vector field $[(\mathrm{u}(p), \mathrm{v}(p))]$ that minimizes the following objective function

$$E(\mathrm{u}, \mathrm{v}) = \iint_\Omega \mu(\mathrm{u}_x^2 + \mathrm{u}_y^2 + \mathrm{v}_x^2 + \mathrm{v}_y^2) + |\nabla f|^2 |(\mathrm{u}, \mathrm{v}) - \nabla f|^2 d\Omega, \quad (5)$$

where $\mathrm{u}_x, \mathrm{u}_y, \mathrm{v}_x, \mathrm{v}_y$ are the spatial derivatives of the field and μ a blending parameter. The objective function consists of two components, namely, a regularization term and a data-driven component. The data-driven component dominates this functional in the object boundaries $[|\nabla f|\ is\ large]$, while the regularization term dictates the functional in areas where the information is constant $[|\nabla f| \rightsquigarrow 0]$. The selected regularization component depends on the orientation of the field that is not natural. More elaborated terms can be used as shown in [7].

As proposed in [25], one can optimize $(\mathrm{u}(p), \mathrm{v}(p))$ through the calculus of variations and a gradient descent method

$$\frac{d\mathrm{u}}{dt} = \mu \underbrace{\nabla^2\mathrm{u}}_{diffusion} - \underbrace{(\mathrm{u} - f_x)|\nabla f|^2}_{data\ attraction}, \frac{d\mathrm{v}}{dt} = \mu \underbrace{\nabla^2\mathrm{v}}_{diffusion} - \underbrace{(\mathrm{v} - f_y)|\nabla f|^2}_{data\ attraction}, \quad (6)$$

where f_x, f_y are the spatial derivatives of f. Such a flow consists of a diffusion component and a data-driven term. The outcome of this PDE will be equivalent to a progressive construction of the GVF starting from the object boundaries and moving toward the flat background. One can observe that the data-driven term is based on the relative change of the edge function f (spatial derivatives). Such a contribution can be considered as a limitation since actual edges do not contribute to the construction of the field. In other words, strong edges as well as weak edges create similar flow due to the diffusion of the flow information. To overcome this issue, we slightly modify the objective function as

$$E(\mathrm{u}, \mathrm{v}) = \iint_\Omega \mu(\mathrm{u}_x^2 + \mathrm{u}_y^2 + \mathrm{v}_x^2 + \mathrm{v}_y^2) + f|\nabla f|^2 |(\mathrm{u}, \mathrm{v}) - \nabla f|^2 d\Omega. \quad (7)$$

Such a modification to our understanding induces to the flow in some sense the ability to overcome weak edges due to noise presence and leads to a fair diffusion of the boundary information where strong edges overcome/compensate flows produced by weak edges.

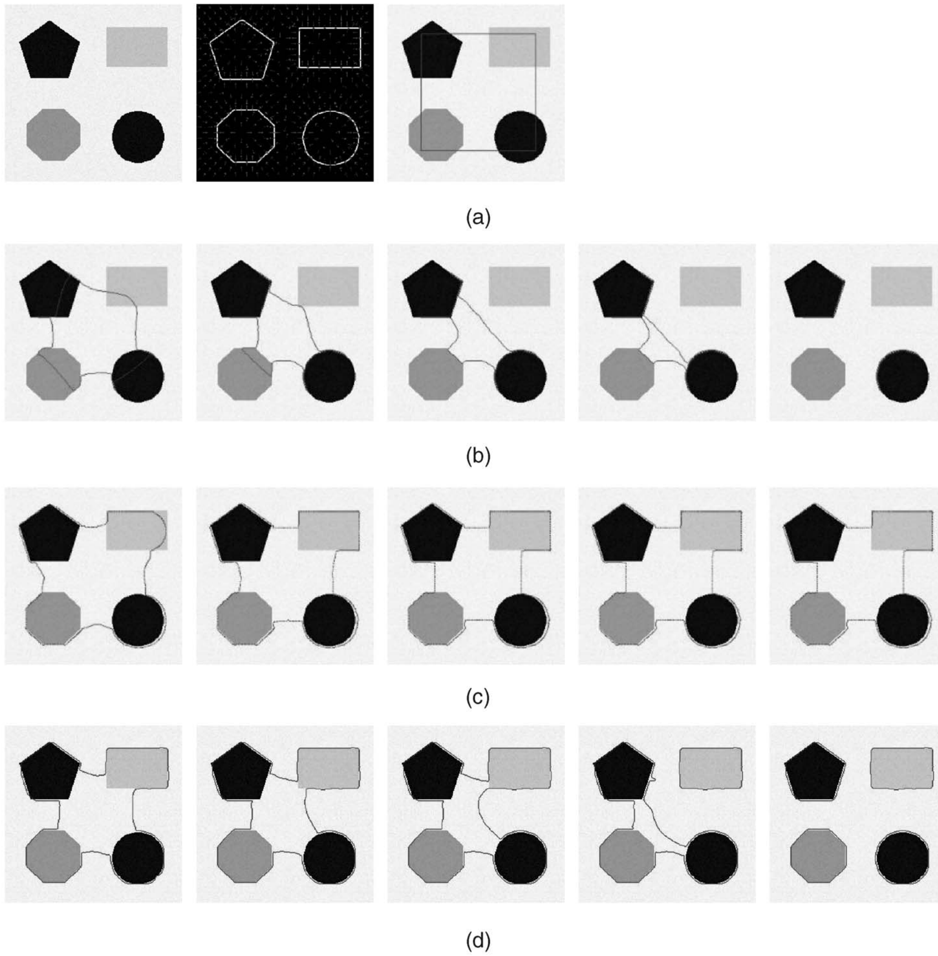


Fig. 1. Boundary extraction by the propagation of curves: (a) Input image, gradient vector flow, and initial curve, (b) geodesic active contour, (c) gradient vector flow snake, and (d) gradient vector flow geometric active contour.

In [25], a geometric flow-based normalized gradient vector field (NGVF) $(\hat{\mathrm{u}}(p) = \mathrm{u}/\sqrt{\mathrm{u}^2+\mathrm{v}^2}, \quad \hat{\mathrm{v}}(p) = \mathrm{v}/\sqrt{\mathrm{u}^2+\mathrm{v}^2})$ was proposed for boundary extraction:

$$C_t(p) = \alpha\ \frac{\partial^2 C}{\partial p^2}(p)\ -\ \beta\frac{\partial^4 C}{\partial p^4}(p) + (\hat{\mathrm{u}}(p), \hat{\mathrm{v}}(p)). \tag{8}$$

Such a flow is relatively free of the initial conditions and can deal with concave regions. However, it depends on the parameterization of the curve, cannot deal with topological changes, and involves second and fourth order derivatives that are difficult to estimate. The objective of our work is to eliminate these shortcomings by integrating the GVF with the geodesic active contour and implementing it using the level set method.

3.2 Integration with the Geodesic Active Contour Flow

As we mentioned earlier, a simplistic interpretation of the NGVF consists of the direction that has to be followed to reach the object boundaries. Therefore, given the current position of the contour, one can assume that the optimal way to deform it locally is in the direction of the NGVF, as proposed in [25]. Evolving the contour in the direction of the normal is a key characteristic of the geodesic active contour flow. Consequently, given this flow, we can claim that optimal propagation of the contour will take place if and only if the unit inward normal and the NGVF are identical. Opposite to that, when the normal is orthogonal to the NGVF, one can consider that evolving the contour in an inappropriate direction is not meaningful. The simplest form of taking these conditions into account is by considering the inner product between the inward unit normal and the NGVF:

$$C_t(p) = ([\hat{\mathrm{u}}, \hat{\mathrm{v}}](p) \cdot \mathcal{N}(p))\mathcal{N}(p). \tag{9}$$

This flow inflates the curve when GVF and the normal are identical, deflates it when GVF and the normal point to opposite directions, and does not deform it when these vectors are orthogonal. Because of the field normalization, boundary information is not considered in an absolute scale. On the other hand, if the field is not normalized, then it will refer to insufficient information to guide the propagation. We recall that, far from the object boundaries, a diffusion process is used to construct the field.

Inspired by the geodesic active contour, one can consider a boundary influence function and modify the proposed geometric flow as follows:

$$C_t(p) = \underbrace{g(p)}_{boundary}\ ([\hat{\mathrm{u}}, \hat{\mathrm{v}}](p) \cdot \mathcal{N}(p))\ \mathcal{N}(p). \tag{10}$$

In the absence of boundary information, the propagation is driven by the inner product between the normal and the NGVF. Such a product can reach the object boundaries from either side and can deal with concave regions. On the other hand, this flow becomes inactive when the curve reaches the vicinity of the object boundaries. During the construction of this flow, we did not consider/impose any internal constraints.

Smoothness constraints are natural when dealing with real objects and are important to guarantee convergence and stability of the contour propagation process. A common selection to impose

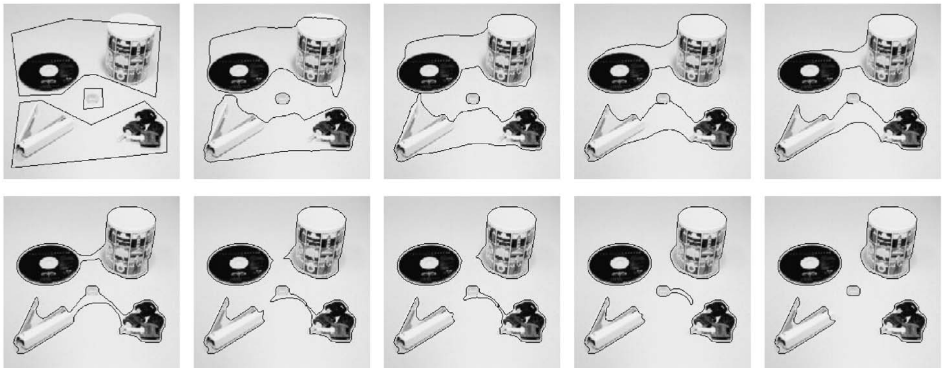


Fig. 2. Gradient vector flow geometric active contours for boundary extraction.

such constraints is via a curvature term. Such a term penalizes the length of the curve as well as the formation of discontinuities:

$$C_t(p) = g(p)\left(\underbrace{\beta\,\mathcal{K}(p)}_{smoothness} + [\hat{\mathrm{u}}, \hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)\right)\mathcal{N}(p), \tag{11}$$

where β balances the contribution between internal constraints and image attraction. The proposed geometric flow has a form similar to the geodesic active contour model motion equation. The term that induces the bidirectionality of the flow is the same as the one used to refine the curve position of the geodesic active contour in the vicinity of the object boundaries. Furthermore, the smoothness constraint is effective during the propagation while it becomes inactive when the curve reaches the object boundaries. This term is identical to the one proposed in the original formulation of the geodesic active contour [3], [11].

According to the original assumptions, the proposed flow does not perform propagation when the NGVF is close to orthogonal to the inward normal. Such a constraint is valid for cases with a single object. On the other hand, when dealing with multiple objects that have parallel line segments, the formations of flow fields that can be orthogonal to the normal (as shown in Fig. 1) is plausible. In that case, propagation will not take place, as well as change of the topology, even if they are supported by the level set technique. The use of regional force applied in the direction of the normal is a standard technique to cope with this limitation. One can consider these region terms as adaptive balloon forces and develop them from the NGVF using the inner product with the normal.

One can determine the magnitude of this force using an exponential function of the absolute value of the inner product: $\left[H(p) = e^{-\delta|[\hat{\mathrm{u}},\hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)|}\right]$, where δ is a scale factor. Such a term has significant values (magnitude) when the normal and the GVF are close to orthogonal. Regarding the direction, again the same product can be used. The sign of this product is an appropriate indicator, leading to the following definition for the adaptive balloon force:[1]

$$H(p) = sign([\hat{\mathrm{u}}, \hat{\mathrm{v}}](p)\cdot\mathcal{N}(p))e^{-\delta|[\hat{\mathrm{u}},\hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)|}.$$

This balloon force can be integrated to the proposed flow and be used when we are in the process of reaching a deadlock in the propagation process. Such a force has to be mutually exclusive with the one derived from the integration of the geodesic active contour and the gradient vector flow;

$$C_t(p) = g(p)\left(\underbrace{\beta\,\mathcal{K}(p)}_{smoothness} + \underbrace{(1-|H(p)|)}_{exclusiveness}\ \underbrace{[\hat{\mathrm{u}}, \hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)}_{boundary} + \underbrace{H(p)}_{balloon}\right)\mathcal{N}(p). \tag{12}$$

The benefit of using this additional balloon force is shown in Figs. 1c and 1d. In the absence of this force, the result we recovered is similar to the one obtained through the use of the original gradient vector flow Fig. 1c. However, it is important to note that this term is beneficial only when dealing with multiple objects. One can consider the level set implementation of this flow;

$$\Phi_t(p) = g(p)((H(p) + \beta\mathcal{K}(p))|\nabla\Phi(p)| - (1-|H(p)|)[\hat{\mathrm{u}}, \hat{\mathrm{v}}](p)\cdot\nabla\Phi(p)). \tag{13}$$

The final flow consists of 1) a term that imposes regularity on the propagation and aims at shrinking the curve toward the object boundaries, 2) a bidirectional flow that moves the curve toward the objects boundaries from either side, and 3) an adaptive bidirectional balloon force that is used to determine the propagation when the boundary term becomes inactive.

The proposed flow can handle a larger set of initial conditions compared to the geodesic active contour and the gradient vector flow fields. However, a more elaborate technique to deal with arbitrary initial conditions is to consider the proposed flow within existing curve-based image segmentation approaches that make use of region-based features. The level set methods [18] were combined with global regional segmentation components leading to techniques that are free of the initial conditions, and can deal with topological changes [4], [20], [27]. These methods exhibit more freedom on the initial conditions compared with the one proposed in this paper.

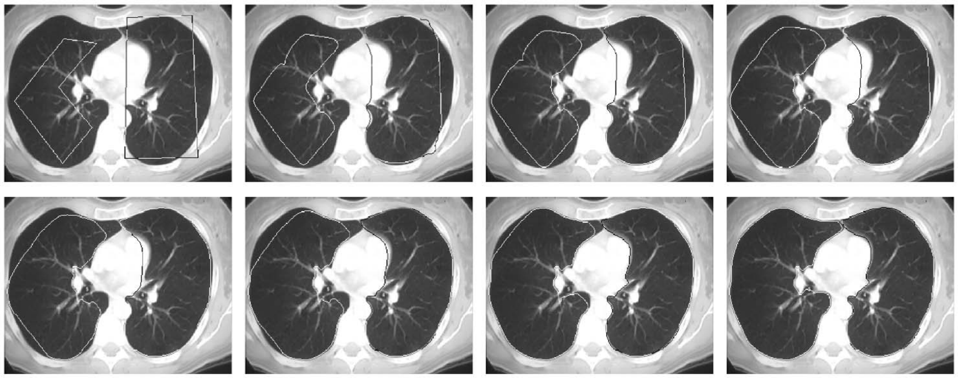
4 Implementation Issues

The main limitation for the use of partial differential equations (and level set methods) in computer vision is poor efficiency due to the fact that classic numerical approximations are pretty unstable, resulting in time-consuming methods. This is due to the need of a small time step that guarantees a stable evolution and convergence to the PDEs. A way to overcome this limitation was introduced in [23] and can be efficiently used to provide a stable numerical method for solving, robustly and rapidly, PDE-based approaches in computer vision. The Additive Operator Splitting schema can be easily applied to implement the level set propagation [8], [14].

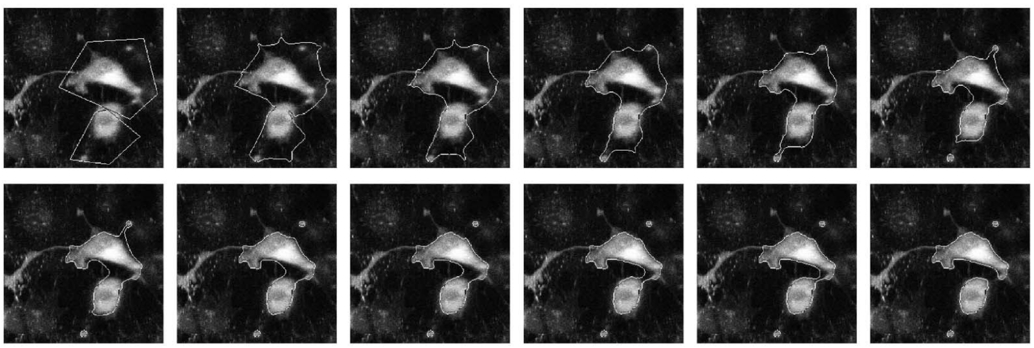
1. A more efficient selection from an implementation point of view consists of the following term:

$$H(p) = \begin{cases} e^{-\delta|[\hat{\mathrm{u}},\hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)|}, & if\ ([\hat{\mathrm{u}}, \hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)) \geq \epsilon \\ -e^{-\delta|[\hat{\mathrm{u}},\hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)|}, & if\ ([\hat{\mathrm{u}}, \hat{\mathrm{v}}](p)\cdot\mathcal{N}(p)) < \epsilon, \end{cases}$$

where ϵ is a positive constant leading close to 0. Such a selection shifts the balloon force toward a deflation nature when applicable.



(a)



(b)

Fig. 3. (a) Boundary extraction using the Gradient Vector Flow Geometric Active Contour for medical images. (b) Bidirectional geometric flows and boundary extraction for microscopic images (the authors would like to acknowledge Charles Kervrann from INRA, Biometry Unit, Jouy-en-Josas, France for providing this image [10]).

To further decrease the required computational cost of the level set propagations, the AOS scheme can be efficiently combined with the Narrow Band Method [1]. The essence of this method is to perform the level set propagation only within a limited band located according to the latest position of the propagating contour. This method requires a frequent reinitialization of the level set function.

4.1 Experimental Results

Real images have been used for the validation of the proposed framework. Promising experimental results (Figs. 2 and 3) were obtained. The ability to change the topology and handle reasonable initial conditions that cannot be dealt with through the use of geodesic active contour is shown in Figs. 2 and 3. As far the computational cost is concerned, we can claim that the approach is close to real time for a 200×200 frame modulo the selection of the initial conditions. Some conceptual comparison between the proposed flow, the original implementation of the gradient vector flow, and the geodesic active contour model is shown in Fig. 1.

5 Conclusions

To summarize, in this paper, we have proposed a geometric boundary-based flow for boundary extraction and image segmentation based on the integration of the gradient vector flow and the geodesic active contour. Such a flow exhibits certain desirable characteristics when compared with the individual components used to derive our approach. It is relatively free of the initial conditions, can deal naturally with topological change, and is implemented using a stable numerical method that has real-time performance.

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