Gradient Vector Flow Fast Geodesic Active Contours

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Abstract

This paper proposes a new front propagation flow for boundary extraction. The proposed framework is inspired by the geodesic active contour model and leads to a paradigm that is relatively free from the initial curve position. Towards this end, it makes use of a recently introduced external boundary force, the gradient vector field that refers to a spatial diffusion of the boundary information. According to the proposed flow, the traditional boundary attraction term is replaced with a new force that guides the propagation to the object boundaries from both sides. This new geometric flow is implemented using a level set approach, thereby allowing dealing naturally with topological changes and important shape deformations. Moreover, the level set motion equations are implemented using a recently introduced numerical approximation scheme, the Additive Operator Splitting Schema (AOS) which has a fast convergence rate and stable behavior. Encouraging experimental results are provided using real images.

1 Introduction

During the last twenty years a wide variety of mathematical and computational frameworks have been proposed to deal with Computer Vision problems. Most of them, are based on the fact that many of the computer vision applications turn out to be image segmentation problems.

More recently, many researchers in the field of computer vision started to pay attention to novel ways of analyzing, formulating and representing the vision problems via variational approaches. One of the most elegant tool of such approaches is the propagation of planar curves which relies on deforming an initial curve towards the lowest potential of a curve-based energy.

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The idea of solving computer vision problems by optimizing curve-based models is quite old [18] and the proposed approaches can be mainly classified into three distinct categories:

- **Deformable Templates**

  This class of approaches is the least flexible one, since it relies on models that are made up of low level geometric components such as lines, circles and ellipses. These models are usually employed to model faces or parts of them (eyes) [10, 15].

- **Point Distribution Models**

  A step further to the rigid and articulated models was the use of specific patterns of variability from a representative training set. Thus, some constraints are introduced on the model parameters that are derived from the distribution of the training set of examples [25]. These ideas were the basis for the point distribution models developed in [7] which is a compromise between rigid models and relatively free deformable models (snakes).

- **Snakes and Active Contour Models**

  A major breakthrough to the curve propagation approaches, the snake model, was made in 1987 by *Kass, Witkin and Terzopoulos* [13]. This model corresponds to an elastic curve that is propagated by image forces towards the minimum of an energy generated by an image. Besides, this model introduces some internal regularization constraints, which ensure the regularity of the curve and limit the bending affect. Inspired by the snake model, a step further was the snake-balloon model [26, 5, 17], the Deformable Template Models [4, 8, 23, 25] and the Geodesic Active Contour [2, 14, 16] that was successfully combined with the level set theory, resulting on a very attractive tool. This model is the basis of our approach.

However, most of these approaches present some considerable limitations that are related with their initial conditions (curve position) and the ability to change the topology of the evolving curve [5, 8, 13, 29]. The first limitation is due to the one direction propagation that is usually imposed by their boundary attraction term (*e.g.* in the direction of the inward normal) [2, 14, 16]. In other words, they aim at either shrinking or expanding (exclusively) the initial curve towards the object boundaries. The second limitation is due to their implementation using Lagrangian approaches that do not allow dealing with topological changes [5, 8, 13, 29].

In this paper, we propose a novel bi-directional boundary-based geometric flow for boundary extraction. The boundary-based information is determined using the gradient vector flow [29] that refers to a diffusion of the gradient boundary space. This diffusion leads to a new external force that drives the propagated curves to the objects boundaries from either side. This new force is used to revise the geodesic active contour model, resulting on an elegant, bi-directional geometric flow. Furthermore, the implementation of this flow is done using level set methods where topological changes are naturally handled. Finally, to deal with the computational complexity induced by the level set methods the Additive Operator Splitting scheme is employed that leads to a fast ["real-time"] gradient flow geodesic active contour model. This scheme was originally proposed in [28] and efficiently used to implement level set propagations [11].

The proposed flow is favorably compared with the original geodesic active contour model [2, 14, 16] because it is free from the initial conditions, and to the gradient vector flow [29] because it can deal naturally with topological changes. Moreover, a novel numerical method is used for its implementation that is extremely fast.

The remainder of this paper is organized as follows: in section 2, the snake as well as the geodesic active contour model are briefly introduced. Section 3 presents the gradient vector flow estimation as well as its elaboration to the geodesic active contour motion equation. In Section 4, we describe the implementation details of our approach. Finally, experimental results and discussion appears in Section 5.

2 Snakes and Active Contours

The classical energy-based snake model has been initially proposed in [13], and successfully applied to deal with a wide variety of Computer Vision Applications. This framework matches a deformable model to an image by means of energy minimization and therefore it exhibits dynamic behavior.

Let $[C : [0,1] \to \mathcal{R}^2,\ p \to C(p)]$ be a parameterized close planar curve and let $I : \mathcal{Z}^+ \times \mathcal{Z}^+ \to \mathcal{R}^+$ be a given image, where we would like to detect the object boundaries. The snake model aims at minimizing the following energy:

$$E[(C)(p)] = \alpha \int_0^1 E_{int}(C(p))dp + \beta \int_0^1 E_{img}(C(p))dp + \gamma \int_0^1 E_{con}(C(p))dp$$

where the internal contour term $[E_{int}]$ constrains the curve to be regular and smooth, the image term $[E_{img}]$ attracts the contour to the desired features, and the constraint term $[E_{con}]$, constrains the solution space. Let us now try to present in a general form the snake energy terms:

- The **internal** energy term stands for the curve regularity $[E_{int}(C(p))]$,

  $$w_{ten}(C(p)) \left| \frac{\partial C}{\partial p} \right|^2 + w_{stif}(C(p)) \left| \frac{\partial^2 C}{\partial p^2}(p) \right|^2$$

  where the first order term makes the snake to behave like a membrane (*i.e.* resists stretching), while the second term makes the snake to act like a thin plate (*i.e.* resists bending).

- The **image** energy term is derived from the observed data, where the snake may be attracted to **lines**, **edges** or **terminations** $[E_{img}(C(p))]$,

  $$w_l E_l(C(p)) + w_e E_e(C(p)) + w_t E_t(C(p))$$

  where $\{w_l, w_e, w_t\}$ are weight constants. In most of the cases, the **line** and **edge** terms are given by

  $$E_l(C(p)) = I(C(p)), \quad E_e(C(p)) = |\nabla I(C(p))|^2$$

  so that if w_l is positive, then the snake is attracted to dark lines and if negative then it is attracted to light lines. The **edge** term attracts the snake to large image gradients (usually w_e is negative). Finally, the **termination** term allows terminations (*i.e.* free ends of lines) or corners to be developed by the snake.

- Finally, the **external** energy is usually derived by some user-defined constraints.

Once an appropriate initialization of the contour is specified, the snake can quickly converge to the nearby energy minimum, using a variational approach.

Numerous provisions have been made in the literature to improve the robustness and stability of the snake model. Towards this direction, in [5], a "balloon force" has been introduced to the snake model. This new term is an anisotropic pressure potential that controls the evolution of the area enclosed by the model and can either inflate or deflate the contour. A step forward was the use of finite elements-based Deformable Template Models to incorporate the prior model information. This information can be either very general such as regularity constraints or very specific such as an exact template [4, 8, 23, 25].

The snake model provides a powerful interactive tool to deal with computer vision problems. However, this approach is "myopic" because of the use of strictly local information and is very sensitive to the initialization step; if a model is initialized too far away from the feature of interest it may fail to locate the appropriate energy minimum. Additionally, this model is strongly dependent on the parameterization of the curve. Moreover, due to the fact that is usually implemented using the Lagrangian approach, it cannot deal naturally with changes of topology which can be considered as a very important drawback. Finally, the last problem of the snake energy relies on the selection of the parameters that determines the contributions of the different energy terms.

2.1 Geodesic Active Contours

The geodesic active contour model [2, 14] was introduced as a geometric alternative for snakes. It can be viewed as an "extension" of classic snakes since it overcomes some of their limitations. A similar model that is geometry-based was proposed in [16].

This model does not impose any rigidity constraints $[w_{stif} = 0]$ and is given by,

$$E[(C)(p)] = \underbrace{\int_0^L g\left(|\nabla I(C(s))|\right) ds}_{Geodesic\ Active\ Contour} = \int_0^1 \underbrace{g\left(|\nabla I(C(p))|\right)}_{attraction\ term} \underbrace{\left|\frac{\partial C}{\partial p}(p)\right|}_{regularity\ term} dp$$

where g a monotonically decreasing function

$$g : [0, +\infty] \to \mathcal{R}^+,\ g(0) = 1, g(x) \to 0 \text{ as } x \to \infty$$

ds is the Euclidean arc-length element, and L the Euclidean length of $C(p)$. In other words, when we try to detect an object we try to find the minimal length geodesic curve that best takes into account the desired image characteristics.

The minimization of the objective function is done using a gradient descent method. Therefore, the initial curve $C_0(.)$ is deformed towards a (local/global) minimum of $E[(C)(p)]$ according to the following equation

$$C_t = \underbrace{g\left(|\nabla I|\right) \mathcal{K}\mathcal{N}}_{boundary\ force} - \underbrace{\left(\nabla g\left(|\nabla I|\right) \cdot \mathcal{N}\right) \mathcal{N}}_{refinement\ force}$$

where t denotes the time as the contour evolves, $\mathcal{N}$ is the inward Euclidean normal, and $\mathcal{K}$ the Euclidean curvature. The above motion equation has a simple interpretation; each point of the contour should move along the normal direction in order to decrease the weighted length of C. There are two forces acting on the contour, both in the direction of the inward normal:

- The first force moves the curve towards the real object boundaries constrained by the curvature effect that ensures regularity during the propagation.
- The second force is applicable only around the real object boundaries $[\nabla g\left(|\nabla I|\right) \neq 0]$ and has a twofold role: (i) it is used to attract the curve to the real boundaries and to overcome along them the propagation constraints imposed by the curvature effect, (ii) it is used as a refinement term that centralizes the curve to the real object boundaries.

The geodesic active contour model is favorably compared with the classical snake due to the fact that it does not depend from the curve parameterization and is relatively free from the initial conditions. However, this model relies to a non-parameterized curve, and evolves an initial curve according to the boundary attraction term towards one direction [inwards or outwards]. Thus, in order to be properly used it demands a *specific* initialization step, where the initial curve should be completely exterior or interior to the real object boundaries.

Many efforts have been made to overcome these shortcomings by introducing some region-based features which make the model free from the initial conditions and more robust [3, 21, 30, 31]. Although these approaches seem to have a reasonable behavior, they still suffer from the one direction flow imposed by the boundary term. In this paper we propose a new geometric flow that overcomes the limitation imposed by the existing boundary-based active contour models.

3 Gradient Vector Flow Active Contours

The Gradient Vector Flow (GVF) framework [29] refers to the definition of a new elegant, bi-directional external force field that captures the object boundaries from either sides and can deal with concave regions. This flow can be considered as an alternative, that is favorably compared, to the distance transform. Opposite to this transform where in most of the cases a binary edge map is required, the GVF is estimated directly from the continuous gradient space. Furthermore, the diffusion process that provides the GVF leads to a measurement that is contextual and not equivalent with the distance from the closest point. This is due to the fact that more that one "boundary" pixels (with different/opposite flows) contribute to the estimation of the GVF.

The first required step within this framework is to determine a continuous edge-based information space which in our case is given by a Gaussian edge detector (zero mean, with σ_E variance) [9],

$$g(p) = \frac{1}{2\pi\sqrt{\sigma_E}} e^{-\frac{|\nabla(G_\sigma * I)(p)|^2}{2\sigma_E^2}},\ \ f(x,y) = 1 - g(p)$$

where $G_\sigma * I$ denotes the convolution of the input image with a Gaussian Kernel (smoothing).

The gradient vector flow [29] consists of a two dimensional vector field $[\mathtt{v}(p) = (u(p), v(p)), p = (x, y)]$ that

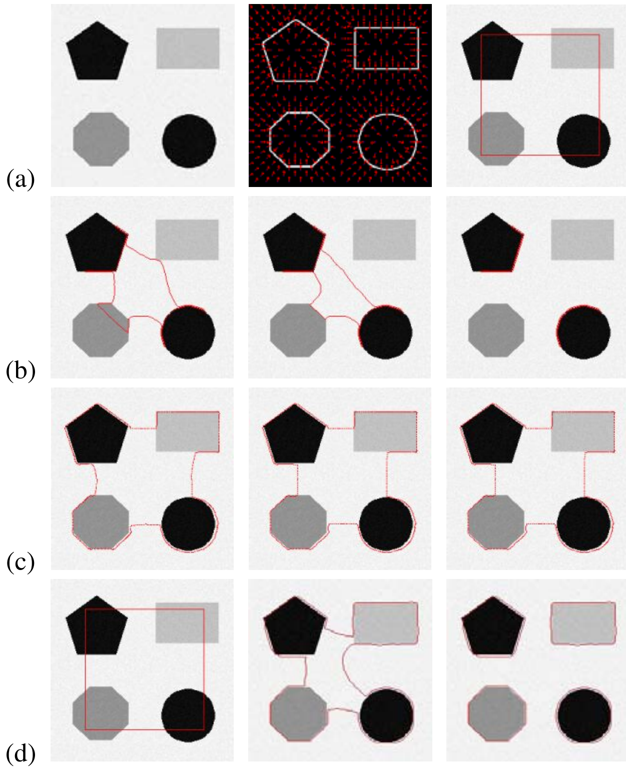

Figure 1. *Boundary extraction by the Propagation of Curves: (**a**) Input Image, Gradient Vector Flow and Initial Curve, (**b**) Geodesic Active Contour, (**c**) Gradient Vector Flow Snake, (**d**) Gradient Vector Flow Geodesic Active Contour.*

minimizes the following energy

$$E(\mathtt{v}) = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |\mathtt{v} - \nabla f|^2 dxdy$$

where μ is a blending parameter. According to this objective function, areas where there the information is constant $[|\nabla f| \rightsquigarrow 0]$, are dominated by the partial derivatives of the vector field, resulting on a smooth flow map. On the other hand, when there are variations on the boundary space $[|\nabla f| \ is \ large]$, the term that dominates the energy is the second one, leading to $\mathtt{v} = \nabla f$. A more detailed interpretation of this energy can be found in [29] which is similar with the one proposed by *Horn and Schunk* [12] for the estimation of optical flow.

However, according to the definition of the objective function, the boundary information is not used directly (only its gradient affects the flow) which might be considered as a drawback. In other words strong edges as well as weak edges create a similar flow due to the diffusion of the flow information. To overcome this weakness we slightly modify the objective function as

$$E(\mathtt{v}) = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + f|\nabla f|^2 |\mathtt{v} - \nabla f|^2 dxdy$$

This important modification induces to the flow the ability to overcome weak edges due to noise presence. Also, it leads to a fair diffusion of the boundary information where strong edges overcome/compensate (in some essence) flows produced by weak edges.

The minimization of the objective function can be easily done using the calculus of variations and the following partial differential equation is obtained

$$\frac{d\mathtt{v}}{dt}(p) = \mu \nabla^2 \mathtt{v}(\mathtt{p}) - f(p)\left(\mathtt{v}(\mathtt{p}) - \nabla f(x,y)\right)|\nabla f(p)|^2$$

The rescaling of the Gradient Vector Flow field $[\hat{\mathtt{v}}(\mathtt{p})]$ leads to a new external force for the boundary term[1]. This field contains mainly contextual (and somehow metric) information [**fig.** (1.a)] and the flow vectors of this field (from direction point of view) point always to the "closest" object boundaries.

Based on the normalized Gradient Vector Flow $[\mathtt{v}/|\mathtt{v}|]$, a new geometric flow has been proposed for boundary extraction in [29], given by

$$C_t(p,t) = \alpha \frac{\partial^2 C}{\partial p^2} - \beta \frac{\partial^4 C}{\partial p^4} + \frac{\mathtt{v}}{|\mathtt{v}|}$$

Although this flow is relatively free from the initial conditions, it cannot deal with topological changes. Moreover, it requires the estimation of the second and fourth order derivative of the curve which cannot be done precisely (numerically) in the discrete space where the propagation is performed.

However, we can note that the modified Gradient Vector Flow field (after the re-scaling) refers to the direction that has to be followed to reach the "closest" object boundaries. Thus, given the latest position of the curve, we can assume that the optimal way to reach these boundaries (from contextual point of view) is to move along the direction of the GVF. Given the fact, that the propagation takes always place in the inward normal direction, it is clear that the optimal propagation is obtained when the GVF and the unit inward normal are identical. On the other hand, the worse case is when the GVF is tangential to the normal. Thus, the best way to determine mainly a contextual (and somehow metric) propagation given the modified GVF and the unit normal is by their inner product as follows,

$$C_t = (\hat{\mathtt{v}} \cdot \mathcal{N})\,\mathcal{N}$$

The interpretation of this flow is clear. When the GVF and the inward normal have the same direction then the flow inflates maximally the curve. On the other hand when these vectors have opposite directions, the flow maximally deflates the curve. Finally, when the GVF is tangent to the normal, then no propagation takes place. However, this motion equation (due to the way that the GVF has been estimated and rescaled) doesn't account for any boundary information. Such information can be easily introduced to the

[1] This operation re-scales the vector field according to the higher norm within the observed GVF norm values in the image.

defined flow as:

$$C_t = g\ (\hat{\mathrm{v}} \cdot \mathcal{N})\ \mathcal{N}$$

The above flow behaves as follows: in the absence of boundary information the propagation is driven by the contextual information. On the other hand, a combination between the contextual information and the strength of the observed edges is used to determine the propagation close to the boundaries. However, this flow does not impose any kind of regularity constraint in the propagation. This is a drawback that can be easily addressed:

$$C_t = \beta\, g \mathcal{K} \mathcal{N} + (1-\beta)\, g\ (\hat{\mathrm{v}} \cdot \mathcal{N})\ \mathcal{N}$$

where β is a constant parameter that balances the contribution between regularity and boundary attraction. It is quite clear that this flow under a constrained scenario, can produce an evolution similar to one obtained by geodesic active contour model. For example, if the term $[\hat{\mathrm{v}} \cdot \mathcal{N}]$ that induces the bi-directionality of the flow is ignored, then flow is comparable to the first term (flow) of the geodesic active contour flow. At the same time, when the propagating curve is located close to the object boundaries, then the flow has the same behavior with the second term of the geodesic active contour flow $[\hat{\mathrm{v}} \cdot \mathcal{N} = \nabla g \cdot \mathcal{N}]$.

However, after a careful review it can be shown that the proposed flow presents a considerable limitation since no propagation is performed when the GVF is tangent (or close to tangent) to the inward normal. This limitation can be dealt with by incorporating an adaptive balloon force. The modified GVF can be used to determine the direction of this new force, which is combined with the boundary force to define the boundary attraction term:

$$C_t = \beta\ g\ \mathcal{K}\ \mathcal{N} + (1-\beta)\ g \left((1-\gamma(\hat{\mathrm{v}} \cdot \mathcal{N}))\ \underbrace{(\hat{\mathrm{v}} \cdot \mathcal{N})}_{attraction} + \gamma(\hat{\mathrm{v}} \cdot \mathcal{N})\ \underbrace{sign(\hat{\mathrm{v}} \cdot \mathcal{N})}_{balloon} \right)\ \mathcal{N}$$

where γ is a zero-mean Laplacian function, of the inner product, between the normal vector and the rescaled GVF,

$$\gamma(\hat{\mathrm{v}} \cdot \mathcal{N}) = \frac{\lambda}{2} e^{-\lambda |\hat{\mathrm{v}} \cdot \mathcal{N}|}$$

The interpretation of the new flow is clear;

- The first force imposes a regularity constraint on the propagation and aims at shrinking the curve towards the object boundaries,
- The second force refers to a sophisticate boundary attraction term that is decomposed in two sub-terms.
  - The first force, is a bidirectional flow that moves the curve towards the objects boundaries from both sides.

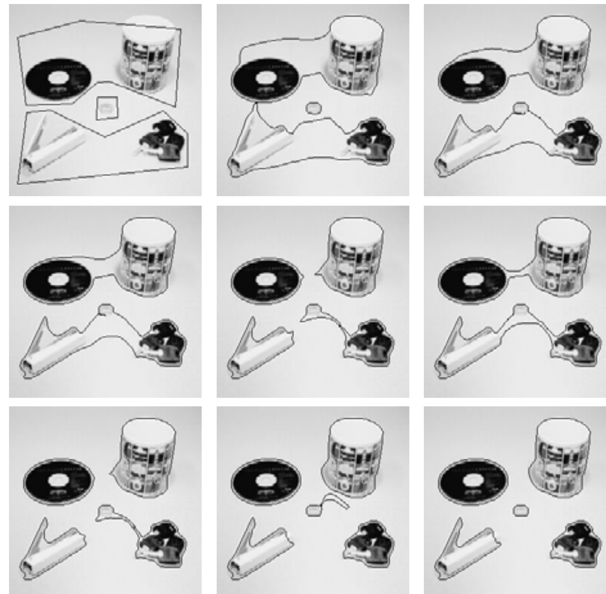

Figure 2. *Gradient Vector Flow Geodesic Active Contour for boundary extraction.*

  - The second sub-force acts as an **adaptive balloon force** $[\mathrm{sign}(\hat{\mathrm{v}} \cdot \mathcal{N})]$ and is activated when the boundary attraction force does not provide enough information to move the curve (the GVF is tangent to the normal $[\hat{\mathrm{v}} \cdot \mathcal{N} \rightsquigarrow 0]$).

These two terms are "mutually exclusive" ($\gamma()$ function).

4 Implementation Issues

The obtained flow can be implemented using a Lagrangian approach, that induces several problems. Thus, it cannot deal with topological changes of the moving front and suffers from instability in the domain of numerical approximations.

This can be avoided by introducing the work of *Osher and Sethian* [20] in our scheme. The central idea is to represent the moving front $\partial R(t)$ as the zero-level set $\{\Phi = 0\}$ of a function Φ. This representation of $\partial R(t)$ is implicit, parameter-free and intrinsic. Additionally, it is topology free. It is easy to show, that if the embedding function Φ deforms according to

$$\Phi_t(p,t) = \mathbf{f}(p)\, |\nabla \Phi(p,t)|$$

then the corresponding moving front evolves according to:

$$C_t(p,t) = \mathbf{f}(p) \mathcal{N}(p)$$

under the condition $[\Phi(C(p,0),0) = 0]$. Surveys regarding the use of level set methods in various domains can be foud at [19, 24].

Thus, the evolution of the proposed flows is equivalent to searching for a steady-state solution of the following equations:

$$\mathbf{Flow3} \ : \Phi_t = \beta \, g \, \mathcal{K} \, |\nabla\Phi| \; - \; (1-\beta) \, g \; (\hat{\mathrm{v}} \cdot \nabla\Phi)$$

$$\mathbf{Flow3} \ : \Phi_t = \beta \, g() \, \mathcal{K} \, |\nabla\Phi| - (1-\beta) \, g() \left(\left(1 - \gamma \left(\hat{\mathrm{v}} \cdot \frac{\nabla\Phi}{|\nabla\Phi|} \right) \right) (\hat{\mathrm{v}} \cdot \nabla\Phi) + \gamma \left(\hat{\mathrm{v}} \cdot \frac{\nabla\Phi}{|\nabla\Phi|} \right) sign \left(\hat{\mathrm{v}} \cdot \frac{\nabla\Phi}{|\nabla\Phi|} \right) \; |\nabla\Phi| \right)$$

The main limitation for the use of partial differential equations (and level set methods) in computer vision is poor efficiency, due to the fact that classic numerical approximations are pretty unstable, resulting on time consuming methods. This is due to the need of a small time step that guarantees a stable evolution and convergence to the PDEs. A way to overcome this limitation was recently introduced in [28] and can be efficiently used to provide a stable numerical method to a wide variety of PDEs (when the necessary conditions are fulfilled). In order to better introduce the AOS, we will consider the one dimension case. Let us considered a diffusion equation of the following form

$$\partial_t u = \mathrm{div} \, (g(|\nabla u|)\nabla u)$$

Then, this diffusion equation can be discretized as follows

$$\partial_t u = \partial_x \, (g(|\partial_u|)\partial_x u)$$

which leads to the following iterative scheme

$$u^{m+1} = [I + \tau A(u^m)] \, u^m$$

which I is the identity matrix and τ is the time step. Although this system updates explicitly the u values using their values from the previous iteration, it is not stable and constrains the time step by an upper bound [28]. A step further can be done using a semi-implicit scheme

$$u^m = [I - \tau A(u^m)] \, u^{m+1}$$

that has a stable behavior but is computationally expensive. Finally, the AOS scheme refers to the following modification of the semi-implicit schema

$$u^{m+1} = [I - \tau A(u^m)]^{-1} \, u^m$$

that has very nice properties. It is very stable, satisfies all the criteria for discrete non-linear diffusion, has low complexity (linear to the number of pixels) and can be easily extended to higher dimensions.

The Additive Operator Splitting schema can be easily applied to implement the level set propagation [11]. In our case, it has a fast convergence rate and a very stable behavior with respect to [flow (3)] which fulfills all the required conditions. On the other hand, the time step has to be significantly decreased for the [flow (3)] in order to guarantee stability. This constraint is imposed by the nature of the flow which does not preserve some of the stability conditions required by the AOS schema.

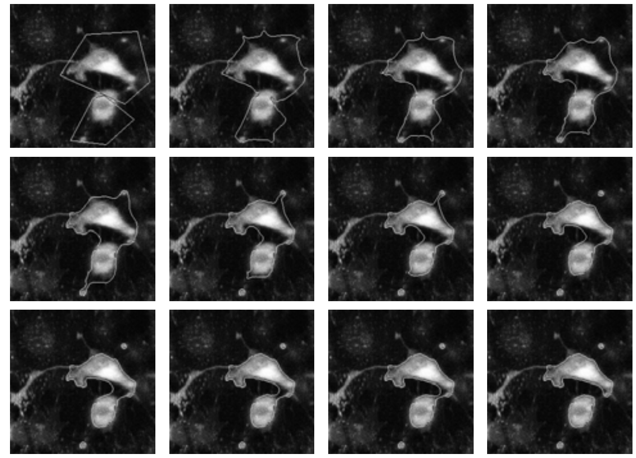

Figure 3. *Gradient Vector Flow Geodesic Active Contour Boundary Extraction in satellite images (Cos and Nisiros islands, Dodecanesa, Greece).*

To further decrease the required computational cost of the level set propagations, the AOS scheme can be efficiently combined with the Narrow Band Method [1] as in [11]. The essence of this method is to perform the level set propagation only within a limited zone that is located around the latest position of the propagating contours (in the inward and outward direction). Thus, the working area is reduced significantly resulting on a significant decrease of the computational complexity per iteration. However, this method requires a frequent re-initialization of the level set functions that is performed using the Fast Marching algorithm [24]. A similar algorithm within the area of automatic control was proposed in [27].

7

Real images have been used for the validation of the proposed framework. Promising experimental results [**fig.** (2,3,4] have been obtained. As far the computational cost is concerned, the average segmentation time for a 200×200 frame using the AOS operator is approximately 1.5 seconds[2].

To summarize, in this paper, we have proposed a novel geometric boundary-based flow for boundary extraction and image segmentation [6]. The boundary information is determined using a modified version of the gradient vector flow and is incorporated to the geodesic active contour model. This modification leads to a bi-directional flow that is relatively free from the initial curve conditions. A level set implementation of this flow leads to a model that can deal naturally with topological changes, while a recently introduced numerical method is used to guarantee stability and fast convergence rate.

Although the proposed flow in most of the cases is favor-

[2]A dual Pentium is used (800 Mhz with, 512 MB Ram)

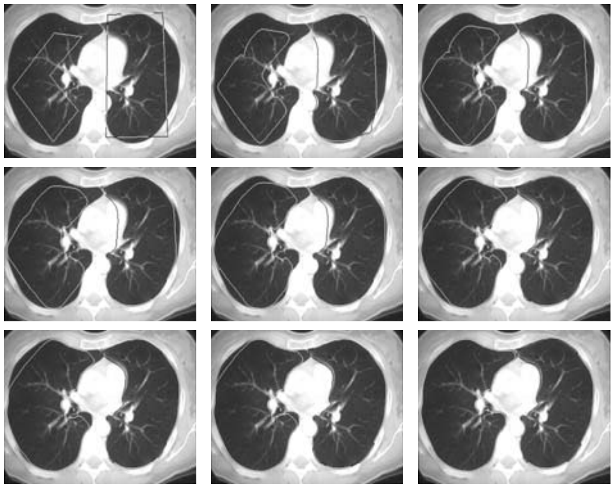

Figure 4. *Boundary extraction using the Gradient Vector Flow Geodesic Active Contour for medical images.*

ably compared with the existing non-parameterized boundary flows (snake, geodesic active contour, gradient vector snake flow) [**fig.** (1)], it is clear that produces optimal results in the case of images with single or convex objects and under the condition that the initial curves contain part of the GVF skeleton. This condition ensures that the propagation will take place in both directions since the curve is propagated always towards the "closest" object boundaries. If this is not the case, then curve would be propagated towards the closest boundary points and will vanish.

To overcome these limitations, the incorporation of the flow to existing curve-based image segmentation approaches (*e.g.* [21]) that make use of region-based features is a short term objective. Also the validation of the framework using a real application - the segmentation of the left ventricle in cardiac images - is a part of the future work [22]. The justification of the model from an energy point of view is also an interesting step.

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