Cooperative Manipulation Exploiting only Implicit Communication

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Abstract— This paper addresses the problem of cooperative object manipulation with the coordination relying solely on implicit communication. We consider a decentralized leader-follower architecture where the leading robot, that has exclusive knowledge of the object's desired trajectory, tries to achieve the desired tracking behavior via an impedance control law. On the other hand, the follower estimates the leader's desired motion via a novel prescribed performance estimation law, that drives the estimation error to an arbitrarily small residual set, and implements a similar impedance control law. Both control schemes adopt feedback linearization as well as load sharing among the robots according to their specific payload capabilities. The feedback relies exclusively on each robot's force/torque, position as well as velocity measurements and apart from a few commonly predetermined constant parameters, no explicit data is exchanged on-line among the robots, thus reducing the required communication bandwidth and increasing robustness. Finally, a comparative simulation study clarifies the proposed method and verifies its efficiency.

I. INTRODUCTION

The study of decentralized multi-robot systems in object carrying tasks (see Fig. 1) has received increasing attention over the last decades. In such tasks, the inter-robot communication has been proven critical, since there is no central unit to supervise the agents' actions. In general, there are two types of communication, namely the explicit and the implicit. The former type is designed solely to convey information such as control signals or sensory data directly to other robots [1]. On the other hand, the latter occurs as a side-effect of robot's interactions with the environment or other robots, either physically (e.g., the interaction forces between the object and the robot) or non-physically (e.g., visual observation). In this case, the information needed is acquired by appropriate sensors attached on the agents.

The most investigated and frequently employed communication form is the explicit one. It usually leads to simpler theoretic analysis and renders teams more effective. However, in case of faulty communication environments, severe problems may arise, such as dropping the object, exertion of excessive forces and performance degradation. Moreover, as the number of cooperating robots increases, communication protocols require complex design to deal with crowded bandwidth. On the other hand, several of the aforementioned limitations can be overcome by employing implicit communication instead. Despite the increased difficulty of the theoretical analysis, it
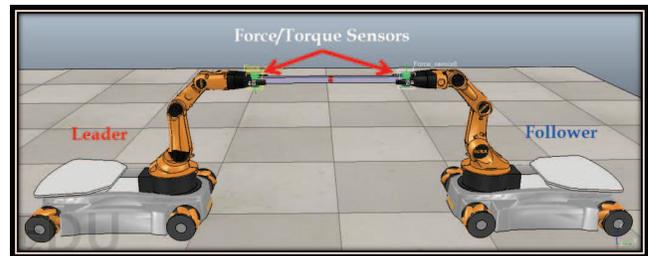
Fig. 1.   Two KUKA Youbots manipulating an object in 3-D motion.

leads to simpler protocols and saves bandwidth as well as power, since no or very few data is explicitly exchanged. It also increases robustness in case of faulty environments as well as stealthiness of operation, since the agent activity is not easily detected.

Cooperative manipulation has been well-studied in the literature, especially the centralized schemes [2]–[5]. Despite its efficiency, centralized control is less robust, since all units rely on a central system, and its complexity increases rapidly as the number of participating robots becomes large. On the other hand, decentralized control usually depends on either explicit communication or off-line knowledge of the desired object trajectory [6]–[8]. Moreover, in other leader-follower schemes [9], [10], the leader has to transmit on-line the desired object trajectory to the follower.

Implicit communication has been exclusively employed in a few decentralized schemes for holonomic mobile manipulators. Kosuge et. al. in a series of works [11]–[13] presented a leader-follower scheme for holonomic manipulators. The leader implements a desired trajectory profile through an impedance scheme, while the follower estimates it through the motion of the object. However, the dynamics of the object are neglected and the estimation error remains bounded close to zero only if the desired acceleration is zero (i.e., trajectories with constant velocity profile). Finally, regarding non-holonomic mobile robots, the follower's passive caster behavior was adopted in [14], [15]. Although, the stability of the follower's contact is established, it is not stated how the object's trajectory can be controlled.

This paper addresses the problem of decentralized cooperative object manipulation. The challenge lies in completely replacing explicit communication with implicit. We employ a leader-follower formation, similarly to [13]. The leader is aware of the object's desired trajectory and implements it via an impedance control law. The follower, estimates it by observing the object's motion and imposes a similar impedance law. Both impedance laws linearize the dynamics, adopt similar control gains and incorporate coefficients for load-sharing. The estimation process is based on the prescribed

performance methodology [16] that drives the estimation error to an arbitrarily small residual set. As a result, the tracking error is ultimately bounded with customizable ultimate bounds. Finally, it should be noted that both agents use solely their own force, position and velocity measurements. The only explicit information needed, is limited down to a few constant parameters, which may be transmitted off-line.

In this work, we extend the current state of art [11]–[13], via a more robust estimation algorithm that converges even though the desired object's acceleration profile is non-zero (i.e., arbitrary object's desired trajectory as long as it is bounded and smooth). Moreover, the customizable ultimate bounds allow us to achieve practical stabilization of the tracking error, with accuracy limited only by the sensors' resolution. Finally, we provide a novel way to share the object load among the participating robots.

The rest of the manuscript is organized as follows: Section II introduces the prescribed performance concept and some preliminary results on dynamical systems. Section III describes the problem and the system's model. The control scheme along with the estimation algorithm are presented in Section IV. Section V validates our approach through simulated paradigms and Section VI concludes the paper. Finally, the proof of the main result is given in the appendix.

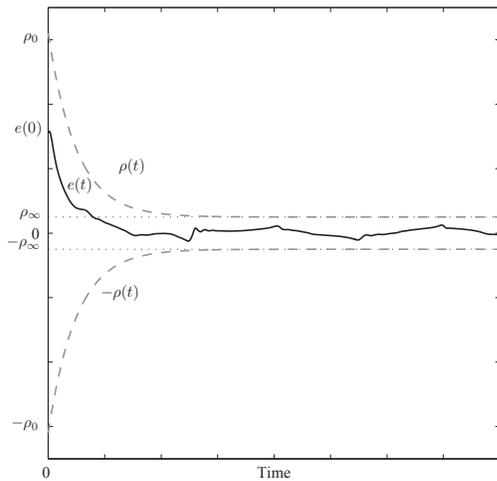## II. DEFINITIONS AND PRELIMINARIES

Fig. 2. Graphical illustration of the prescribed performance definition.

A. Prescribed Performance

It will be clearly demonstrated in the sequel that the concepts and techniques of prescribed performance control, recently proposed in [17], [18] for nonlinear systems, are innovatively adapted herein to develop a novel estimation scheme. Prescribed performance characterizes the behavior where an error converges to a predefined arbitrarily small residual set with convergence rate no less than a certain predefined value. In that respect, consider a generic scalar error $e(t)$. The mathematical expression of prescribed performance is given by the following inequalities:

$$-\rho(t) < e(t) < \rho(t), \ \forall t \geq 0 \qquad (1)$$

where $\rho(t)$ is a smooth and bounded function of time satisfying $\lim_{t \to \infty} \rho(t) > 0$, called performance function. The aforementioned statements are clearly illustrated in Fig. 2 for an exponential performance function $\rho(t) = (\rho_0 - \rho_\infty) e^{-st} + \rho_\infty$ with appropriately chosen positive constants ρ_0, ρ_∞, s, which impose transient and steady state performance characteristics on the error $e(t)$.

B. Dynamical Systems

Consider the initial value problem:

$$\dot{\xi} = h(t, \xi), \ \xi(0) = \xi^0 \in \Omega_\xi \qquad (2)$$

with $h : \Re_+ \times \Omega_\xi \to \Re^n$ where $\Omega_\xi \subset \Re^n$ is a non-empty open set.

Definition 1: [19] A solution $\xi(t)$ of the initial value problem (2) is maximal if it has no proper right extension that is also a solution of (2).

Theorem 1: [19] Consider the initial value problem (2). Assume that $h(t, \xi)$ is: a) locally Lipschitz on ξ for almost all $t \in \Re_+$, b) piecewise continuous on t for each fixed $\xi \in \Omega_\xi$ and c) locally integrable on t for each fixed $\xi \in \Omega_\xi$. Then, there exists a maximal solution $\xi(t)$ of (2) on the time interval $[0, \tau_{\max})$, such that $\xi(t) \in \Omega_\xi, \forall t \in [0, \tau_{\max})$.

Proposition 1: [19] Assume Theorem 1 holds. For a maximal solution $\xi(t)$ on the time interval $[0, \tau_{\max})$ with $\tau_{\max} < \infty$ and for any compact set $\Omega'_\xi \subset \Omega_\xi$ there exists a time instant $t' \in [0, \tau_{\max})$ such that $\xi(t') \notin \Omega'_\xi$.
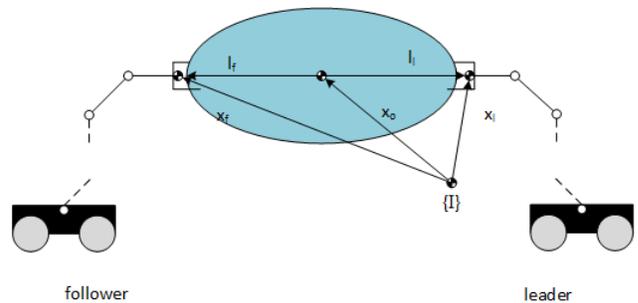
III. PROBLEM FORMULATION

Fig. 3. Two mobile manipulators handling a rigidly grasped object.

Consider two mobile manipulators in a leader-follower scheme handling a rigidly grasped object as shown in Fig. 3. We assume that each robot has at least 6 DoFs and is fully actuated. Only the leader knows the object's desired trajectory $x_{dl}(t)$, whereas the follower estimates it by $x_{df}(t)$ via its own state measurements. Owing to the strict communication constraints (i.e., no on-line communication), the problem becomes very challenging. Hence, we relax the asymptotic tracking requirements down to ultimate bound-edness of the tracking errors. We assume that measurements of position, velocity and interaction forces/torques with the object are available for each robot exclusively. The geometric and inertial parameters of the mobile manipulators as well as of the object are considered known. Finally, the only information allowed to be exchanged, is the values of a few

constant control parameters that are transmitted off-line to the follower.

A. Kinematics

We denote the leader's and follower's task space (end effector) coordinates with respect to an inertial frame $\{I\}$ by $x_i = \left[x_{ip}^T, x_{ir}^T\right]^T$, $i \in \{l, f\}$, where x_{ip} and x_{ir} correspond to the end-effector's position and orientation respectively. Similarly, we denote the object's coordinates with respect to $\{I\}$ by $x_o = \left[x_{op}^T, x_{or}^T\right]^T$. Let also q_i, $i \in \{l, f\}$ be the joint space variables. Invoking the forward kinematics equations, we express the task space variables as a nonlinear function of the joint variables: $x_i = f_i(q_i)$, $i \in \{l, f\}$. Differentiating, we also obtain: $\dot{x}_i = J_i(q_i)\dot{q}_i$, $i \in \{l, f\}$, where $J_i(q_i) = \frac{\partial f_i(q_i)}{\partial q_i}$ is the Jacobian matrix. Moreover, since the contacts are considered rigid, the following relations hold:

$$x_{ip} = x_{op} + l_i, \ x_{ir} = x_{or} + a_i, \ i \in \{l, f\} \qquad (3)$$

where the vectors $l_i = [l_{ix}, l_{iy}, l_{iz}]^T$ and $a_i = [a_{ix}, a_{iy}, a_{iz}]^T$ represent the relation between the object's and the end effector's frames (see Fig. 3). Since the object's geometric parameters are considered known, each robot may compute the object's coordinates via (3). Furthermore, we establish a velocity relation by differentiating (3) as follows:

$$\dot{x}_i = J_{oi}\dot{x}_o, \ i \in \{l, f\} \qquad (4)$$

where J_{oi} is the Jacobian from the end-effector to the object's center of mass. Since the end-effector and the object are rigidly connected, the aforementioned Jacobian has always full rank and hence a well defined inverse J_{oi}^{-1}. Thus, each robot may compute the velocity of the object's center of mass through (4). Finally, differentiating once more, we establish the acceleration relation:

$$\ddot{x}_i = \dot{J}_{oi}\dot{x}_o + J_{oi}\ddot{x}_o, \ i \in \{l, f\} \qquad (5)$$

B. Dynamics

The dynamic model in terms of task space coordinates, for a single robot, is described by:

$$M_i(q_i)\ddot{x}_i + C_i(\dot{q}_i, q_i)\dot{x}_i + G_i(q_i) = U_i + F_i, \ i \in \{l, f\} \quad (6)$$

where M_i is the positive definite inertial matrix, C_i is a matrix representing Coriolis and centrifugal forces and G_i represents gravitational forces. The vector F_i, $i \in \{l, f\}$ represents the interaction force/torque exerted at the end effector by the object and U_i, $i \in \{l, f\}$ denotes the input task space wrench. The relation between the joint torques τ_i and the task space wrench is given by: $\tau_i = \overline{J}_i^T U_i + \left(I - J_i^T \overline{J}_i^T\right)\tau_{in}$, $i \in \{l, f\}$, where $\overline{J}_i$ is the generalized inverse that is consistent with the equations of motion of the manipulator [3]. The vector τ_{in} does not contribute to the end-effector's wrench and can be regulated independently to achieve secondary goals (e.g., manipulability increase or collision avoidance). Invoking the kinematic relations (3)-(5), we may express the aforementioned dynamic models (6) with respect to the object's coordinates as follows:

$$M_{oi}(q_i)\ddot{x}_o + C_{oi}(\dot{q}_i, q_i)\dot{x}_o + G_{oi}(q_i) = J_{oi}^T U_i + J_{oi}^T F_i \quad (7)$$

for $i \in \{l, f\}$, where

$$M_{oi}(q_i) = J_{oi}^T M_i(q_i) J_{oi}, \ G_{oi}(q_i) = J_{oi}^T G_i(q_i),$$
$$C_{oi}(\dot{q}_i, q_i) = J_{oi}^T (C_i(\dot{q}_i, q_i) J_{oi} + M_i(q_i)\dot{J}_{oi})$$

Similarly, the dynamic equation of the object is given by:

$$M_o(x_o)\ddot{x}_o + C_o(\dot{x}_o, x_o)\dot{x}_o + G_o(x_o) = F_o \qquad (8)$$

Assuming that no other external forces are exerted on the object, the total force F_o equals to $F_o = -J_{ol}^T F_l - J_{of}^T F_f = -GF$, where $G = \left[J_{ol}^T, J_{of}^T\right]$ denotes the grasp matrix of the overall configuration and $F = \left[F_l^T, F_f^T\right]^T$.

Remark 1: Wrenches that lie on the null space of the grasp matrix G do not contribute to the object dynamics. Therefore, we may incorporate in the control scheme an extra component $F_{int,i} = \left(I - G^{\#}G\right)\hat{F}_{int}$, $i \in \{l, f\}$, that belongs to the null space of G, in order to regulate the steady-state internal forces, where $G^{\#}$ is the right pseudo-inverse of G. Since l_i, $i \in \{l, f\}$ are considered known to both agents, if $\hat{F}_{int}$ is constant, no communication is needed during task execution in order to compute G, $G^{\#}$ and $F_{int,i}$.

IV. Control Methodology

A. Impedance Control Scheme

The inertial and geometric parameters of both mobile manipulators and the object are considered known, hence a feedback linearization scheme may be applied in each robot. In this respect, we select the following control inputs:

$$U_i = -F_i + J_{oi}^{-T}\left(M_{oi}V_i + C_{oi}\dot{x}_o + G_{oi}\right), \ i \in \{l, f\} \quad (9)$$

to cancel the nonlinearities of (7). Moreover, the auxiliary inputs V_i, $i \in \{l, f\}$ are chosen as:

$$V_i = \ddot{x}_{cmd,i} + M_o^{-1}J_{oi}^T\left(F_i - F_{di}\right), \ i \in \{l, f\} \qquad (10)$$

imposing thus the desired impedance behavior [1] $\ddot{x}_o = \ddot{x}_{cmd,i} + M_o^{-1}J_{oi}^T\left(F_i - F_{di}\right)$ where F_{di}, $i \in \{l, f\}$ denote the desired robot/object interaction wrench:

$$F_{di} = F_{int,i} - J_{oi}^{-T}c_i\left(C_o\dot{x}_o + G_o + M_o\ddot{x}_{cmd,i}\right) \qquad (11)$$

Notice that the aforementioned selection cancels the object's nonlinearities, ensures adequate internal forces via $F_{int,i}$ (see Remark 1) and achieves the motion control objectives through the appropriate design of $\ddot{x}_{cmd,i}$, that will be presented in the sequel. Moreover, the load distribution coefficients c_i, $i \in \{l, f\}$, that are subject to the design constraints $c_l + c_f = 1$, $c_l, c_f > 0$, are assigned values according to the payload capabilities of the mobile manipulators (e.g., in case of heterogeneous robots) thus introducing a load sharing attribute as opposed to previous related work [11]–[13][2]. Finally, the commanded acceleration signal, that is responsible for the tracking objective, is designed as follows:

$$\ddot{x}_{cmd,i} = \ddot{x}_{di} - D_i\left(\dot{x}_o - \dot{x}_{di}\right) - K_i\left(x_o - x_{di}\right) \qquad (12)$$

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[1]The desired impedance behaviour can be easily verified by substituting (9) and (10) in (7).

[2]In these works, the object's dynamics was neglected.

where D_i, K_i, $i \in \{l, f\}$ are diagonal positive definite control gain matrices. As stated above, $x_{dl}(t)$ and $x_{df}(t)$ stand for the actual object's desired trajectory profile to be implemented by the leader, and its estimate at the follower's side respectively. Hence, substituting (9)-(12) in (7), we obtain the leader's and follower's tracking error dynamics:

$$\Delta \ddot{x}_i + D_i \Delta \dot{x}_i + K_i \Delta x_i = M_o^{-1} J_{oi}^T (F_i - F_{di}) \quad (13)$$

where $\Delta x_i = x_o - x_{di}$, $i \in \{l, f\}$. Selecting $D_l = D_f = D$ and $K_l = K_f = K$ as well as adding the object's dynamics (8) in (13), we get:

$$\Delta \ddot{\overline{x}} + D \Delta \dot{\overline{x}} + K \Delta \overline{x} = 0 \quad (14)$$

where $\Delta \overline{x} = x_o - \frac{(c_l+1)x_{dl} + (c_f+1)x_{df}}{3}$. In this way, the positive definiteness of the control gain matrices D, K renders the aforementioned system asymptotically stable. Therefore, $\Delta \overline{x}$ and $\Delta \dot{\overline{x}}$ converge exponentially to the origin (i.e., $\Delta \overline{x}(\Delta \overline{x}(0), \Delta \dot{\overline{x}}(0); t) \overset{\exp}{\to} 0$ and $\Delta \dot{\overline{x}}(\Delta \overline{x}(0), \Delta \dot{\overline{x}}(0); t) \overset{\exp}{\to} 0$). Finally, it can be easily verified that the object's trajectory suffices:

$$x_o(t) = \frac{(c_l+1)x_{dl}(t) + (c_f+1)x_{df}(t)}{3}$$
$$+ \Delta \overline{x}(\Delta \overline{x}(0), \Delta \dot{\overline{x}}(0); t) \quad (15)$$

B. Estimation law

The follower is not aware of the object's actual desired trajectory profile $x_{dl}(t)$. However, even though explicit communication between the leader and the follower is not permitted, the follower may estimate the error $x_{dl}(t) - x_{df}(t)$ by measuring the term $3\frac{x_o(t) - x_{df}(t)}{c_l+1}$, which is easily obtained via (15), after a few trivial algebraic manipulations, and the fact that $\Delta \overline{x} \overset{\exp}{\to} 0$. Moreover, the estimator should also compensate for acceleration residuals, since acceleration measurements are not available. Thus, we sacrifice asymptotic stability by adopting a robust prescribed performance estimator that guarantees ultimate boundedness of the estimation error $x_{dl}(t) - x_{df}(t)$ and consequently ultimate boundedness of the tracking error $x_o(t) - x_{df}(t)$.

Let us define the error $e(t) = 3\frac{x_o(t) - x_{df}(t)}{c_l+1}$. The expression of prescribed performance for each element of $e(t) = [e_1(t), e_2(t), \ldots]^T$ is given by the following inequalities:

$$-\rho_j(t) < e_j(t) < \rho_j(t), \forall t \geq 0 \quad (16)$$

where $\rho_j(t)$ denotes the corresponding performance function. As stated in Subsection II-A, a candidate performance function would be $\rho_j(t) = (\rho_{j,0} - \rho_{j,\infty})e^{-st} + \rho_{j,\infty}$ where the constant s dictates the exponential convergence rate, $\rho_{j,\infty}$ denotes the ultimate bound and $\rho_{j,0}$ is chosen to satisfy $\rho_{j,0} > |e_j(0)|$. Hence, following the prescribed performance control technique [17], the estimation law is designed:

$$\dot{x}_{dfj} = k_j \ln \left(\frac{1 + \frac{e_j(t)}{\rho_j(t)}}{1 - \frac{e_j(t)}{\rho_j(t)}} \right), k_j > 0 \quad (17)$$

from which the follower's estimate $x_{dfj}(t)$ is calculated via a simple integration. Moreover, differentiating (17) with

respect to time, we acquire the desired acceleration signal:

$$\ddot{x}_{dfj} = \frac{2k_j}{1 - \left(\frac{e_j(t)}{\rho_j(t)}\right)^2} \frac{\dot{e}_j(t)\rho_j(t) - e_j(t)\dot{\rho}_j(t)}{\rho_j^2(t)} \quad (18)$$

which is bounded if the performance bounds (16) are met.

C. Stability Analysis

The main results of this work are summarized as follows.
Theorem 2: Consider the error $e(t) = [e_1(t), e_2(t), \ldots]^T = 3\frac{x_o(t) - x_{df}(t)}{c_l+1}$. Given a smooth and bounded desired trajectory $x_{dl}(t)$ with bounded derivatives as well as some appropriately selected performance functions $\rho_j(t)$ satisfying $|e_j(0)| < \rho_j(0)$, the estimation law (17) guarantees that $|e_j(t)| < \rho_j(t), \forall t \geq 0$.

Corollary 1: The follower's estimation error and the object's trajectory tracking error are ultimately bounded.

Proof: Notice from Theorem 2 and (15) that:

$$|e_j(t)| = \left| x_{dlj}(t) - x_{dfj}(t) + \frac{3\Delta \overline{x}_j(\Delta \overline{x}_j(0), \Delta \dot{\overline{x}}_j(0); t)}{c_l+1} \right| < \rho_j(t)$$

which leads to:

$$|x_{dlj}(t) - x_{dfj}(t)| < \rho_j(t) + \frac{3|\Delta \overline{x}_j(\Delta \overline{x}_j(0), \Delta \dot{\overline{x}}_j(0); t)|}{c_l+1} \quad (19)$$

Therefore, the estimation error $|x_{dlj}(t) - x_{dfj}(t)|$ is ultimately bounded by $\rho_{j,\infty} \equiv \lim_{t \to \infty} \rho_j(t)$ owing to the fact that $\Delta \overline{x}_j(\Delta \overline{x}_j(0), \Delta \dot{\overline{x}}_j(0); t) \overset{\exp}{\to} 0$. Similarly:

$$|x_{oj}(t) - x_{dlj}(t)| < \frac{c_f+1}{3}|x_{dlj}(t) - x_{dfj}(t)|$$
$$+ |\Delta \overline{x}_j(\Delta \overline{x}_j(0), \Delta \dot{\overline{x}}_j(0); t)|$$

Therefore, the tracking error $|x_{oj}(t) - x_{dlj}(t)|$ is ultimately bounded by $\frac{c_f+1}{3}\rho_{j,\infty}$. ∎

Remark 2: The aforementioned ultimate bounds depend directly on $\rho_{j,\infty}$, which can be set arbitrarily small to a value reflecting the resolution of the measurement device, thus achieving practical convergence of the estimation and tracking errors to zero. Moreover, the transient response depends on the convergence rate of the performance functions $\rho_j(t)$, that is affected by the parameter s, as well as the choice of the impedance control gain matrices D, K in (14).

Remark 3: The proposed method does not utilize any explicit on-line communication. The only information needed on-line to implement the developed control schemes concerns the measurements acquired exclusively by each robot's sensor suite (i.e., force, position and velocity). Some constant parameters, though, should be transmitted off-line, namely the gain matrices D, K, the load distribution coefficients c_i, $i \in \{l, f\}$, the internal force $\hat{F}_{int}$ and the contact points relative to the object. Nevertheless, this amount of information is not significant.

Remark 4: The proposed estimation scheme is more robust against desired trajectory profiles with non-zero acceleration than previous works presented in [11]–[13]. The only necessary condition concerns the smoothness and boundedness of the desired trajectory. In this sense, our method guarantees bounded closed loop signals and practical asymptotic stabilization of the estimation and tracking errors.
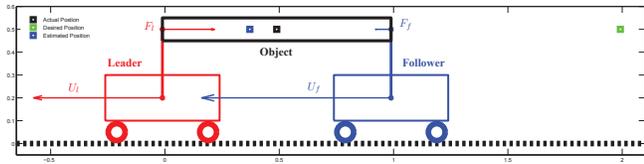
V. SIMULATIONS

Fig. 4. Two mobile robots handling an object in 1-D motion. The leader knows the object's desired trajectory. The follower estimates it via (17).

We consider a simple 1-D scenario with two mobile robots in a leader-follower scheme handling an object (see Fig. 4). The leader is assigned the desired sinusoidal trajectory and the follower estimates it via the proposed algorithm (17), by simply observing the motion of the object and without communicating explicitly with the leader. A comparative simulation study was carried out between the proposed control scheme and the one presented in [11], assuming that the object load is equally shared to both agents, i.e., $c_l = c_f = 0.5$. Moreover, in order to examine the robustness of the closed loop system, we considered a realistic case, where the model parameters and the force measurements adopted in both control schemes deviate up to 5% from their actual values. Finally, the damping and stiffness coefficients were selected as $D = 2$, $K = 1$, the parameters of the proposed estimator and the one presented in [11] were chosen as $k_1 = 0.5$, $\rho_1(t) = 0.49e^{-t} + 0.01$ and $a = 2$, $b = 1$ respectively and $F_{int,i}$, $i \in \{l, f\}$ were set to zero.
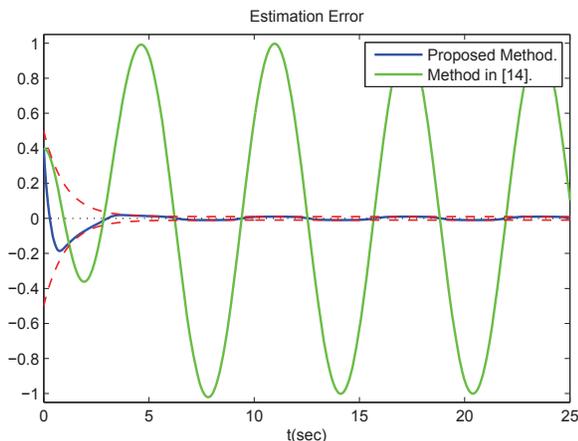

Fig. 5. The estimation errors along with the performance bounds imposed by the proposed method.

The results of the comparative simulation study are given in Figs. 5-7. Notice that both the estimation error (Fig. 5) and the tracking error (Fig. 6) of the proposed scheme practically converge to zero without requesting high control input signals (see Fig. 7). On the contrary, the method presented in [11] was unable to control the system satisfactorily and yielded high control signals owing to the non-zero acceleration profile of the desired trajectory. Finally, the accompanying video demonstrates the aforementioned
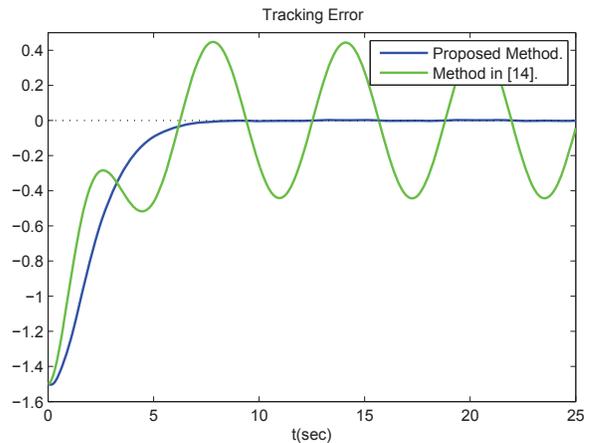

Fig. 6. The tracking errors.

comparative simulation study as well as a realistic simulated paradigm of the proposed method with two KUKA Youbots manipulating an object in a 3-D motion (see Fig. 1), carried out in the Virtual Robot Experimentation Platform (V-REP).

VI. CONCLUSION

This paper presented a leader-follower scenario for cooperative object manipulation under implicit communication. We managed to completely avoid explicit on-line communication. The only information exchanged off-line concerned the values of a few constant parameters. The leader imposed the object's desired trajectory profile via an impedance scheme. The follower adopted a similar impedance law with identical control gains and a prescribed performance estimator to evaluate the object's desired trajectory, that was unaware of. The achieved ultimate boundedness of the estimation errors resulted in ultimate boundedness of the tracking errors, with bounds depending exclusively on the choice of certain designer-specified performance parameters, thus enabling practical stabilization. We extended the related literature by: i) introducing the object's dynamics, ii) incorporating a load sharing technique and iii) robustifying the estimation process against any smooth and bounded object's desired trajectory. Future research efforts will be devoted towards extending the current methodology in multiple cooperating robots and considering uncertainties in the dynamic model of both the mobile manipulators and the grasped object.

VII. APPENDIX

Proof of Theorem 2: The proof follows identical arguments for each element of $e(t)$. First, let us define the normalized error:

$$\xi_j = \frac{e_j(t)}{\rho_j(t)}. \tag{20}$$

Differentiating ξ_j with respect to time, we obtain a dynamical system of the form $\dot{\xi}_j = h_j(t, \xi_j)$. We also define the non-empty and open set $\Omega_{\xi_j} = (-1, 1)$. Since $|e_j(0)| < \rho_j(0)$, we conclude that $\xi_j(0) \in \Omega_{\xi_j}$. Additionally, owing to the smoothness of: a) the leader's desired trajectory, b) the exponentially decreasing term $\Delta \dot{\bar{x}}_j$ and c) the proposed estimation scheme (17) over Ω_{ξ_j}, the function $h_j(t, \xi_j)$ is
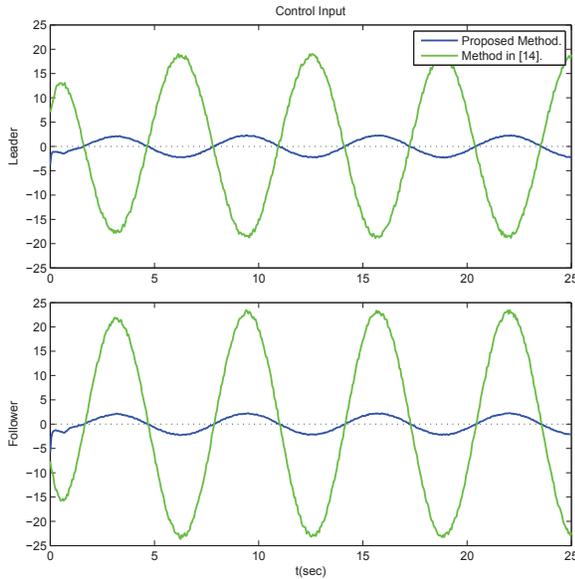
Fig. 7. The control input signals U_l and U_f.

continuous for all $t \geq 0$ and $\xi_j \in \Omega_{\xi_j}$. Therefore, the hypotheses of Theorem 1 stated in Subsection II-B hold and the existence of a maximal solution $\xi_j(t)$ on a time interval $[0, \tau_{\max})$ such that $\xi_j(t) \in \Omega_{\xi_j}$, $\forall t \in [0, \tau_{\max})$ is ensured.

Notice now that the transformed error signal $\varepsilon_j(t) = \ln\left(\frac{1+\xi_j(t)}{1-\xi_j(t)}\right)$ is well defined for all $t \in [0, \tau_{\max})$. Hence, consider the positive definite and radially unbounded function $V_j = \frac{1}{2}\varepsilon_j^2$. Differentiating, we obtain:

$$\dot{V}_j = \frac{2\varepsilon_j}{(1-\xi_j^2)\rho_j(t)}\left(\dot{x}_{dlj}(t) + 3\frac{\Delta\dot{\bar{x}}_j\left(\Delta\bar{x}_j(0),\Delta\dot{\bar{x}}_j(0);t\right)}{c_l+1}\right.$$
$$\left. - k_j\varepsilon_j - \xi_j\dot{\rho}_j(t)\right) \quad (21)$$

Since $\Delta\dot{\bar{x}}_j\left(\Delta\bar{x}_j(0),\Delta\dot{\bar{x}}_j(0);t\right)$ is exponentially decreasing, $\xi_j \in \Omega_{\xi_j}$ and $\dot{x}_{dlj}(t), \dot{\rho}_j(t)$ are bounded either by assumption or by construction, we conclude that:

$$\left|\dot{x}_{dlj}(t) + 3\frac{\Delta\dot{\bar{x}}_j\left(\Delta\bar{x}_j(0),\Delta\dot{\bar{x}}_j(0);t\right)}{c_l+1} + \xi_j\dot{\rho}_j(t)\right| \leq \bar{U}_j$$

for an unknown positive constant $\bar{U}_j$. Moreover, $\frac{1}{1-\xi_j^2} > 1$, $\forall\xi_j \in \Omega_{\xi_j}$ and $\rho_j(t) > 0$, $\forall t \geq 0$. Hence, we conclude that $\dot{V}_j < 0$ when $|\varepsilon_j(t)| > \frac{\bar{U}_j}{k_j}$ and consequently that:

$$|\varepsilon_j(t)| \leq \bar{\varepsilon}_j = \max\left\{|\varepsilon_j(0)|, \frac{\bar{U}_j}{k_j}\right\}, \forall t \in [0, \tau_{\max}). \quad (22)$$

Thus, taking the inverse logarithmic function we get:

$$-1 < \frac{e^{-\bar{\varepsilon}_j}-1}{e^{-\bar{\varepsilon}_j}+1} = \underline{\xi}_j \leq \xi_j(t) \leq \overline{\xi}_j = \frac{e^{\bar{\varepsilon}_j}-1}{e^{\bar{\varepsilon}_j}+1} < 1. \quad (23)$$

Therefore, $\xi_j(t) \in \Omega'_{\xi_j} = \left[\underline{\xi}_j, \overline{\xi}_j\right]$, $\forall t \in [0, \tau_{\max})$, which is a nonempty and compact subset of Ω_{ξ_j}. Now we will prove by contradiction that $\tau_{\max}$ can be replaced with ∞. Assuming $\tau_{\max} < \infty$ and since $\Omega'_{\xi_j} \subset \Omega_{\xi_j}$, Proposition 1 in Subsection II-B dictates the existence of a time instant $t' \in [0, \tau_{\max})$ such that $\xi_j\left(t'\right) \notin \Omega'_{\xi_j}$, which is a clear contradiction. Therefore, $\tau_{\max}$ is extended to ∞. As a result,

all closed loop signals remain bounded and moreover $\xi_j(t) \in \Omega'_{\xi_j} \subset \Omega_{\xi_j}$, $\forall t \geq 0$. Finally, from (20) and (23), we conclude:

$$-\rho_j(t) < \underline{\xi}_j\rho_j(t) \leq e_j(t) \leq \overline{\xi}_j\rho_j(t) < \rho_j(t)$$

for all $t \geq 0$, which completes the proof.

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