Robust Model-Free Formation Control with Prescribed Performance for Nonlinear Multi-Agent Systems

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***Abstract*— In this paper, we consider the formation control problem for multi-agent systems with unknown nonlinearities and disturbances, under an undirected communication protocol. Exploiting the recently developed prescribed performance control methodology, a robust distributed control scheme of minimal complexity is proposed that achieves and maintains arbitrarily fast and accurately the desired formation. No information regarding the agents' dynamic model is employed in the design procedure. Moreover, contrary to the related works on multi-agent systems, the transient and steady state response is fully decoupled by the underlying graph topology, the control gains selection and the agents' model uncertainties. In particular, the achieved performance is a priori and explicitly imposed by certain designer-specified performance functions. Finally, the theoretical findings are clarified and verified by an extensive simulation study.**

I. INTRODUCTION

During the last two decades, distributed cooperative control of multi-agent systems has received considerable attention (see the seminal works [1]–[5] for example) owing to a wide range of applications it is involved in, e.g., transportation, sensor networks, aerial, mobile and underwater robotics, multi-point surveillance, etc. In particular, the leader-follower scheme, according to which the following agents aim at reaching a consensus with the leader's state employing only locally available information, has become very popular, since in the absence of any central control system and without global coordinate information, following a leader is a reasonable motivation.

Although the majority of the works on distributed cooperative control considers known and simple dynamic models, there exist, however, many practical engineering systems which cannot be modeled accurately and which are constantly subject to environmental disturbances. Thus, taking into account the inherent model uncertainties when designing robust distributed control schemes is of paramount importance. On the other hand, extending towards this direction the existing nominal control schemes, becomes a very challenging task on account of the increasing design complexity by the interacting system dynamics as reflected by the local intercourse specifications. Nevertheless, multi-agent consensus/synchronization/formation control for systems with unknown nonlinear dynamics and disturbances was studied in [6]–[13] employing neuro/fuzzy approximating structures to compensate for the model uncertainties.

Unfortunately, the aforementioned approaches inherently introduce certain issues affecting closed loop stability and robustness. Specifically, even though the existence of a closed loop initialization set as well as of control gain values that guarantee closed loop stability is proven, the problem of proposing an explicit constructive methodology capable of a priori imposing the required stability properties is not addressed. As a consequence, the produced control schemes yield inevitably reduced levels of robustness against modeling imperfections. Moreover, the results are restricted to be local as they are valid only within the compact set where the capabilities of the universal approximators hold. Furthermore, the introduction of approximating structures increases the complexity of the proposed control schemes in the sense that extra adaptive parameters have to be updated (i.e., nonlinear differential equations have to be solved numerically) and extra calculations have to be conducted to output the control signal, thus making implementation difficult.

Another important issue associated with distributed cooperative control schemes for multi-agent systems under model uncertainties, concerns the transient and steady state response of the closed loop system. Traditionally, the synchronization/consensus error is proven to convergence within a residual set, whose size depends on control design parameters and some unknown, (though bounded), terms. However, no systematic procedure exists to accurately compute the required upper bounds, thus making the a priori selection of the aforementioned control parameters to satisfy certain steady state behavior, practically impossible. Moreover, the transient behavior (i.e., convergence rate) is difficult to establish analytically as it is affected heavily by the agents' model dynamics and the status of the overall underlying interaction topology, both of which are considered unknown.

In this work, we consider a generic class of first order nonlinear multi-agent systems, under an undirected communication protocol. More specifically, we design a decentralized model-free formation control scheme in the sense that each agent utilizes only local relative state information from its neighborhood set to calculate its own control signal, without incorporating any prior knowledge of the model nonlinearities/disturbances or any approximation structures to acquire such knowledge, thus bypassing the aforementioned stability and robustness issues in approximation based control schemes. Additionally, the transient and steady state response is solely determined by certain designer-specified

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This work was supported by the EU funded project RECONFIG: Cognitive, Decentralized Coordination of Heterogeneous Multi-Robot Systems via Reconfigurable Task Planning, FP7-ICT-600825, 2013-2016.

performance functions and is fully decoupled by the agents' dynamic model, the underlying graph topology and the control gains selection, which further relaxes the control design procedure. Finally, the proposed methodology results in a low complexity design. Actually, it is a static scheme involving very few and simple calculations to output the control signal, thus making its distributed implementation straightforward.

II. Problem Statement

Consider a multi-agent group of a leader and N followers, with the leading agent acting as an exosystem that generates a desired command/reference trajectory for the multi-agent group. The following agents, which have to be controlled, obey a first order nonlinear dynamic model, described as follows:

$$\dot{x}_i = f_i(x_i) + g_i(x_i)\, u_i + d_i(t)\,,\ i = 1, \ldots, N \qquad (1)$$

where $x_i \in \Re$, $i = 1, \ldots, N$ denote the state of each agent, $f_i, g_i : \Re \to \Re$, $i = 1, \ldots, N$ are unknown locally Lipschitz vector fields, $u_i \in \Re$, $i = 1, \ldots, N$ are the control inputs and $d_i : \Re_+ \to \Re$, $i = 1, \ldots, N$ represent piecewise continuous and bounded external disturbance terms. Additionally, we assume that the control gain functions $g_i(x_i)$, $i = 1, \ldots, N$ are strictly positive (or negative), i.e., there exist positive constants $g_i^\star$ such that $|g_i(x_i)| \geq g_i^\star > 0$, $\forall x_i \in \Re$, $i = 1, \ldots, N$, which is a sufficient controllability condition for the aforementioned nonlinear dynamics. Without loss of generality we assume that all $g_i(x_i)$, $i = 1, \ldots, N$ are strictly positive.

An undirected graph $G = (V, E)$ is used to model the communication among the agents, where $V = \{v_1, \ldots, v_N\}$ denotes the set of vertices that represent each agent. The set of edges is denoted as $E \subseteq V \times V$ and the graph is assumed to be simple, i.e. $(v_i, v_i) \notin E$ (there exist no self loops). The adjacency matrix associated with the graph G is denoted as $A = [a_{ij}] \in \Re^{N \times N}$ with $a_{ij} \in \{0, 1\}$, $i, j = 1, \ldots, N$. If $a_{ij} = 1$ then the agent i obtains information regarding the state of the j-th agent (i.e., $(v_i, v_j) \in E$), whereas if $a_{ij} = 0$ then there is no state-information flow from agent j to agent i (i.e., $(v_i, v_j) \notin E$). Furthermore, the set of neighbors of a vertex v_i is denoted by $N_i = \{v_j : (v_i, v_j) \in E\}$ and the degree matrix is defined as $D = \mathsf{diag}([D_i]) \in \Re^{N \times N}$ with $D_i = \sum_{j \in N_i} a_{ij}$. Moreover, since the graph is undirected, the adjacency is a mutual relation, thus A is symmetric, i.e., $a_{ij} = a_{ji}$, $\forall i, j = 1, \ldots, N$. In this respect, the Laplacian matrix of the graph, which is denoted as $L = D - A \in \Re^{N \times N}$, is also symmetric. Additionally, the state of the leader node (labeled v_0) is given by $x_0 : \Re_+ \to \Re$ and is further assumed to be smooth and bounded. However, the desired command/reference trajectory information is only provided to a subgroup of the N agents. The access of the following agents to the leader's state is modeled by a diagonal matrix $B = \mathsf{diag}([b_1, b_2 \ldots, b_n]) \in \Re^{N \times N}$. If b_i, $i \in \{1, 2, \ldots, N\}$ is equal to 1, then the i-th agent obtains state-information from the leader node; otherwise if b_i, $i \in \{1, 2, \ldots, N\}$ equals to 0, then the i-th agent cannot obtain state-information from the leader node. Thus, we may also define the augmented graph as $\bar{G} = (\bar{V}, \bar{E})$, where $\bar{V} = V \cup \{v_0\}$ and $\bar{E} = E \cup \{(v_i, v_0) : b_i = 1\} \subseteq \bar{V} \times \bar{V}$ as well as the augmented set of neighbors $\bar{N}_i = \{v_j : (v_i, v_j) \in \bar{E}\}$, $i = 1, \ldots, N$.

In the sequel, we formulate the robust formation control problem with prescribed transient and steady state performance for the aforementioned multi-agent system, that will be confronted herein. More specifically, we aim at designing a distributed control protocol for the following agents, considering unknown model nonlinearities and external disturbances, such that they create arbitrarily fast and maintain with arbitrary accuracy a constant feasible (in the sense of [14]) formation, described by the desired relative offsets c_{ij}, $i = 1, \ldots, N$ with respect to each member $j \in \{0, 1, \ldots, N\}$ of their augmented communication set $\bar{N}_i$. Apparently, when no relative offsets are required (i.e., all $c_{ij} = 0$, $i = 1, \ldots, N$), we consider the synchronization/consensus problem, where a common response, dictated by the leader node, is required for all following agents. Finally, to solve the aforementioned multi-agent control problem, the following assumption is required on the graph topology.

Assumption: The communication graph G is connected and there exists at least one $b_i \neq 0$, $i \in \{1, 2, \ldots, N\}$.

Remark 1: The aforementioned assumption dictates that $L + B$ is an irreducibly diagonally dominant M-matrix [15]. An M-matrix is a square matrix having its off-diagonal entries non-positive and all principal minors nonnegative, thus $L + B$ is positive definite [15]. Moreover, it should be noticed that the following common graph topologies can be considered as special cases of this assumption:

- The graph G contains a spanning tree and at least the root node can get access to the leader node.
- The augmented graph $\bar{G}$ is itself a spanning tree with the root being the leader node.
- The augmented graph $\bar{G}$ has an hierarchical structure.

III. Control Scheme

Since we consider only local relative state information, the control law of each agent should be based on the neighborhood error feedback:

$$e_i = \sum_{j \in N_i} a_{ij}(x_i - x_j + c_{ij}) + b_i(x_i - x_0 + c_{i0}) \qquad (2)$$

for $i = 1, \ldots, N$, where c_{ij}, $i = 1, \ldots, N$ and $j \in \bar{N}_i$, denote the relative offsets that define the desired feasible formation. Moreover, let us also define the overall neighborhood error vector as $\bar{e} = [e_1, \ldots, e_N]^T \in \Re^N$, which, employing the graph topology and after some trivial algebraic manipulations, becomes:

$$\bar{e} = (L + B)(\bar{x} - \bar{x}_0 + \bar{c}) \qquad (3)$$

where $\bar{x} = [x_1, \ldots, x_N] \in \Re^N$ is the overall state vector of the multi-agent system, $\bar{x}_0 = [x_0, \ldots, x_0]^T \in \Re^N$ and

$$\bar{c} = (L+B)^{-1} \begin{bmatrix} \sum_{j \in N_1} a_{1j} c_{1j} + b_1 c_{10} \\ \vdots \\ \sum_{j \in N_N} a_{Nj} c_{Nj} + b_N c_{N0} \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} \tag{4}$$

denotes the relative offset c_i of the i-th agent, $i = 1, \ldots, N$ with respect to the leader, as dictated by the desired formation. In this way, the desired formation is expressed with respect to the leader state, thus it is achieved when the state x_i of each agent approaches the leader state x_0 with the corresponding offset c_i, $i = 1, \ldots, N$. Hence, defining the disagreement formation variable as $\bar{\delta} = [\delta_1, \ldots, \delta_N]^T = \bar{x} - \bar{x}_0 + \bar{c}$, the formation control problem is solved if the disagreement errors δ_i, $i = 1, \ldots, N$ enter arbitrarily fast into an arbitrarily small neighborhood of the origin. However, the disagreement formation variables δ_i, $i = 1, \ldots, N$ are global quantities and thus cannot be measured distributively based on the local intercourse specifications, as they involve information directly from the leader as well as from the whole graph topology via employing the inverse of $L + B$ in (4). Nevertheless, observing (3) and utilizing the positive definiteness of $L+B$ (as mentioned in Remark 1), we obtain:

$$\|\bar{\delta}\| \leq \frac{\|\bar{e}\|}{\lambda_{\min}(L+B)} \tag{5}$$

from which we conclude that the neighborhood error $\bar{e}$ may represent a valid metric of the formation quality. In this respect, transient and steady state bounds imposed on the neighborhood errors e_i, $i = 1, \ldots, N$ can be directly translated into actual performance bounds on the disagreement formation variables δ_i, $i = 1, \ldots, N$. Thus, selecting for each neighborhood error e_i, $i = 1, \ldots, N$ the corresponding performance functions $\rho_i(t) = (\rho_{i0} - \rho_\infty) e^{-lt} + \rho_\infty$, $i = 1, \ldots, N$ [16] such that: i) $|e_i(0)| < \rho_{i0}$, $i = 1, \ldots, N$ and ii) the parameters l, ρ_∞ incorporate the desired transient and steady state specifications respectively, it can be easily verified that the solution of the prescribed performance control problem for all neighborhood errors e_i, $i = 1, \ldots, N$ leads directly to the solution of the formation control problem confronted herein. More specifically, notice from (5) that guaranteeing $|e_i(t)| < \rho_i(t)$, $\forall t \geq 0$, $i = 1, \ldots, N$, imposes explicitly exponential convergence with rate l of the disagreement errors δ_i, $i = 1, \ldots, N$ to the residual set $\Delta = \left\{ \delta \in \Re : |\delta| \leq \frac{\rho_\infty}{\lambda_{\min}(L+B)} \right\}$. Although $\lambda_{\min}(L+B)$ is a global topology variable and thus cannot be employed in distributed control schemes, instead we might either use a conservative lower bound $4\sin^2\left(\frac{\pi}{4N+2}\right) \leq \lambda_{\min}(L+B)$ as presented in [17], that depends on the number of agents N and not on the graph topology, or alternatively run initially a distributed connectivity estimation algorithm (based either on power iteration or on spectral analysis), similarly to [18], [19], to identify $\lambda_{\min}(L+B)$. In any case, given a steady state error specification $\delta_{\max}$ for the disagreement formation variables, the performance parameter ρ_∞ can be selected as:

$$\rho_\infty = 4\sin^2\left(\frac{\pi}{4N+2}\right)\delta_{\max} \tag{6}$$

in the conservative case or $\rho_\infty = \hat{\lambda}\delta_{\max}$, where $\hat{\lambda}$ is the result of the distributed connectivity estimation algorithm.

In the sequel, the concepts and techniques in the scope of the prescribed performance control methodology, recently proposed in [20] for nonlinear systems, are innovatively adapted to deal with the formation control problem in multi-agent systems. In this way, we propose a distributed control protocol, without incorporating any information on the model nonlinearities or the external disturbances, that guarantees $|e_i(t)| < \rho_i(t)$, $\forall t \geq 0$, $i = 1, \ldots, N$ and leads consequently to the solution of the robust formation control problem with prescribed performance for the considered multi-agent system.

Theorem 1: Given the neighborhood errors e_i, $i = 1, \ldots, N$ as defined in (2) and the appropriately selected corresponding performance functions $\rho_i(t)$, $i = 1, \ldots, N$, the distributed control scheme:

$$u_i(e_i, t) = -k_i \ln\left(\frac{1 + \frac{e_i}{\rho_i(t)}}{1 - \frac{e_i}{\rho_i(t)}}\right) \text{ with } k_i > 0 \tag{7}$$

for $i = 1, \ldots, N$ guarantees $|e_i(t)| < \rho_i(t)$, $\forall t \geq 0$ for $i = 1, \ldots, N$ as well as the boundedness of all closed loop signals.

To prove our concept, we first define the normalized neighborhood error vector:

$$\xi = [\xi_1, \ldots, \xi_N]^T := \left[\frac{e_1}{\rho_1(t)}, \ldots, \frac{e_N}{\rho_N(t)}\right]^T \triangleq (\rho(t))^{-1}\bar{e}, \tag{8}$$

where $\rho(t) = \mathsf{diag}([\rho_1(t), \ldots, \rho_N(t)])$. Differentiating ξ with respect to time, we obtain:

$$\dot{\xi} = (\rho(t))^{-1}\left(\dot{\bar{e}} - \dot{\rho}(t)\xi\right). \tag{9}$$

Moreover, differentiating (3) with respect to time and substituting the dynamics of the agents from (1) as well as the control input (7), we get:

$$\dot{\bar{e}} = (L+B)\left(f(\bar{x}) + g(\bar{x})K\varepsilon(\xi) + d(t) - \dot{\bar{x}}_0(t)\right), \tag{10}$$

where

$$\begin{aligned} f(\bar{x}) &= [f_1(x_1), \ldots, f_N(x_N)]^T \\ g(\bar{x}) &= \mathsf{diag}([g_1(x_1), \ldots, g_N(x_N)]) \\ K &= \mathsf{diag}([k_1, \ldots, k_N]) \\ \varepsilon(\xi) &= \left[\ln\left(\frac{1+\xi_1}{1-\xi_1}\right), \ldots, \ln\left(\frac{1+\xi_N}{1-\xi_N}\right)\right]^T \\ d(t) &= [d_1(t), \ldots, d_N(t)]^T. \end{aligned}$$

Furthermore, notice from (3) and (8) that

$$\bar{x}(\xi, t) := \bar{x}_0(t) - \bar{c} + (L+B)^{-1}\rho(t)\xi. \tag{11}$$

Thus, substituting (10) in (9), the closed loop dynamical system of ξ may be written as:

$$\begin{aligned}\dot{\xi} &= h(t,\xi)\\ &= (\rho(t))^{-1}\left[(L+B)\left(f(\bar{x}(\xi,t)) - g(\bar{x}(\xi,t))K\varepsilon(\xi)\right.\right.\\ &\quad \left.\left.+ d(t) - \dot{\bar{x}}_0(t)\right) - \dot{\rho}(t)\xi\right]. \end{aligned} \tag{12}$$

Finally, let us also define the open set:

$$\Omega_\xi = \underbrace{(-1,1)\times\cdots\times(-1,1)}_{N \text{ times}}.$$

In what follows, we proceed in two phases. First, the existence of a unique maximal solution $\xi(t)$ of (12) over the set Ω_ξ for a time interval $[0,\tau_{\max})$ (i.e., $\xi(t)\in\Omega_\xi$, $\forall t\in[0,\tau_{\max})$) is ensured. Then, we prove that the proposed control scheme (7) guarantees, for all $t\in[0,\tau_{\max})$: a) the boundedness of all closed loop signals of (12) as well as that b) $\xi(t)$ remains strictly within a compact subset of Ω_ξ, which leads by contradiction to $\tau_{\max}=\infty$ and consequently to the completion of the proof.

Phase A. The set Ω_ξ is nonempty and open. Moreover, the performance functions $\rho_i(t)$, $i=1,\ldots,N$ have been selected to satisfy $\rho_i(0) > |e_i(0)|$, $i=1,\ldots,n$. As a consequence, $\left|\frac{e_i(0)}{\rho_i(0)}\right| < 1$ which results in $\xi(0)\in\Omega_\xi$. Additionally, h is continuous on t and locally Lipschitz on ξ over the set Ω_ξ. Therefore, the hypotheses of Theorem 54 in [21] (pp.476) hold and the existence of a maximal solution $\xi(t)$ of (12) on a time interval $[0,\tau_{\max})$ such that $\xi(t)\in\Omega_\xi$, $\forall t\in[0,\tau_{\max})$ is ensured.

Phase B. We have proven in Phase A that $\xi(t)\in\Omega_\xi$, $\forall t\in[0,\tau_{\max})$ and more specifically that:

$$\xi_i(t) = \tfrac{e_i(t)}{\rho_i(t)} \in (-1,1),\; i=1,\ldots,n \tag{13}$$

for all $t\in[0,\tau_{\max})$, from which we obtain that $e_i(t)$ is absolutely bounded by $\rho_i(t)$, $i=1,\ldots,n$ for all $t\in[0,\tau_{\max})$. Similarly, we may prove from (11), that:

$$\bar{x}(\xi,t) \equiv \left(\bar{x}_0(t) - \bar{c} + (L+B)^{-1}\rho(t)\xi\right) \in \Omega_x \tag{14}$$

for all $t\in[0,\tau_{\max})$, with:

$$\Omega_x = \left\{x\in\Re^N : \|x\| \le \sup_{t\ge 0}\{\|\bar{x}_0(t)-\bar{c}\|\} + \tfrac{\sqrt{N}\min\{\rho_i(0)\}}{\lambda_{\min}(L+B)}\right\} \tag{15}$$

denoting a compact set, owing to the boundedness of $\bar{x}_0(t)$ and $\rho(t)$ by assumption and by construction respectively. Furthermore, owing to (13), the error vector:

$$\varepsilon(\xi) = \left[\ln\left(\frac{1+\xi_1}{1-\xi_1}\right),\ldots,\ln\left(\frac{1+\xi_N}{1-\xi_N}\right)\right]^T \tag{16}$$

is well defined for all $t\in[0,\tau_{\max})$. Thus, differentiating with respect to time the positive definite and radially unbounded function $V_\varepsilon = \frac{1}{2}\varepsilon^T\varepsilon$ and substituting (12), we obtain:

$$\begin{aligned}\dot{V}_\varepsilon = \varepsilon^T r(\rho(t))^{-1}&\left[(L+B)\left(f(\bar{x}(\xi,t))\right.\right.\\ &\left.\left.- g(\bar{x}(\xi,t))K\varepsilon(\xi) + d(t) - \dot{\bar{x}}_0(t)\right) - \dot{\rho}(t)\xi\right]\end{aligned}$$

where $r := \nabla_\xi\varepsilon = \mathrm{diag}\left(\left[\frac{2}{1-\xi_1^2},\ldots,\frac{2}{1-\xi_N^2}\right]\right)$ is positive definite owing to (13). Exploiting: i) the Extreme Value Theorem, owing to (14) and the fact that $f(\cdot)$ is continuous and Ω_x is compact, as well as ii) the boundedness of $d(t)$, $\dot{\bar{x}}_0(t)$ and $\dot{\rho}(t)$, there exists a positive constant $\bar{F}$ independent of $\tau_{\max}$, such that:

$$\|(L+B)(f(\bar{x}(\xi,t)) + d(t) - \dot{\bar{x}}_0(t)) - \dot{\rho}(t)\xi\| \le \bar{F}$$

for all $t\in[0,\tau_{\max})$. Notice further that the matrices r, $(\rho(t))^{-1}$ and $(L+B)$ are positive definite. Moreover, by assumption the diagonal matrix $g(\bar{x}(\xi,t))$ is also positive definite. Therefore, $\dot{V}_\varepsilon < 0$ when $\|\varepsilon(t)\| > \frac{\bar{F}}{\lambda_{\min}(L+B)\min\{k_i g_i^\star\}}$. Thus, we conclude that:

$$\|\varepsilon(t)\| \le \bar{\varepsilon} = \max\left\{\|\varepsilon(0)\|, \tfrac{\bar{F}}{\lambda_{\min}(L+B)\min\{k_i g_i^\star\}}\right\} \tag{17}$$

for all $t\in[0,\tau_{\max})$, which, by taking the inverse logarithmic function in (16), leads to:

$$-1 < \tfrac{e^{-\bar{\varepsilon}}-1}{e^{-\bar{\varepsilon}}+1} = \underline{\xi} \le \xi_i(t) \le \bar{\xi} = \tfrac{e^{\bar{\varepsilon}}-1}{e^{\bar{\varepsilon}}+1} < 1,\; i=1,\ldots,N \tag{18}$$

for all $t\in[0,\tau_{\max})$. Furthermore, the control signals (7) remain also bounded:

$$|u_i(e_i,t)| \le \bar{u}_i = k_i\bar{\varepsilon},\; i=1,\ldots,N$$

for all $t\in[0,\tau_{\max})$.

Up to this point, what remains to be shown is that $\tau_{\max}$ can be extended to ∞. In this direction, notice by (18) that $\xi(t)\in\Omega_\xi'$, $\forall t\in[0,\tau_{\max})$, where the set:

$$\Omega_\xi' = \underbrace{[\underline{\xi},\bar{\xi}]\times\cdots\times[\underline{\xi},\bar{\xi}]}_{N \text{ times}}$$

is a nonempty and compact subset of Ω_ξ. Hence, assuming $\tau_{\max}<\infty$ and since $\Omega_\xi'\subset\Omega_\xi$, Proposition C.3.6 in [21] (pp. 481) dictates the existence of a time instant $t'\in[0,\tau_{\max})$ such that $\xi\left(t'\right)\notin\Omega_\xi'$, which is a clear contradiction. Therefore, $\tau_{\max}=\infty$. Thus, all closed loop signals remain bounded and moreover $\xi(t)\in\Omega_\xi'\subset\Omega_\xi$, $\forall t\ge 0$. Finally, multiplying (18) by $\rho_i(t)$, we also conclude:

$$-\rho_i(t) < \underline{\xi}\rho_i(t) \le e_i(t) \le \bar{\xi}\rho_i(t) < \rho_i(t),\; i=1,\ldots,N \tag{19}$$

for all $t\ge 0$ and consequently the solution of the robust formation control problem with prescribed performance for the considered multi-agent system.

Remark 2: The proposed control protocol for the generic class of multi-agent systems considered herein, is decentralized in the sense that each agent utilizes only local relative state information from its neighborhood set to calculate its own control signal. Moreover, it does not incorporate any prior knowledge of the model nonlinearities/disturbances. Additionally, no approximation structures (i.e., neural networks, fuzzy systems, etc.) have been employed to acquire such knowledge. Furthermore, the proposed methodology results in a low complexity design. Notice that no hard calculations (neither analytic nor numerical) are required to output the proposed control signal, thus making its distributed implementation straightforward.

Remark 3: From the aforementioned proof it can be deduced that the proposed control scheme achieves its goals without residing on the need of rendering $\bar{\varepsilon}$ (see (17)) arbitrarily small, by adopting extreme values of the control gains k_i, $i = 1, ..., N$. More specifically, notice that (18) and consequently (19), which encapsulates the prescribed performance notion, hold no matter how large the finite bound $\bar{\varepsilon}$ is. In the same spirit, large model uncertainties can be compensated for, as they affect only the size of $\bar{\varepsilon}$ through $\bar{F}$, but leave unaltered the achieved stability properties. Hence, the actual performance, which is solely determined by the performance functions $\rho_i(t)$, $i = 1, \ldots, N$ becomes isolated against model uncertainties, extending thus greatly the robustness of the proposed control scheme. Furthermore, unlike what is the common practice in the related literature (i.e., the control gains are tuned towards satisfying a desired performance, nonetheless without any a priori guarantees), the selection of the control gains k_i, $i = 1, ..., N$ is significantly simplified to adopting those values that lead to reasonable control effort (i.e., low gain values may lead to oscillatory behavior within the performance bounds which is improved when increasing them with the expense of higher control effort). Finally, it should be noticed that contrary to the standard distributed control schemes for static connected graphs, whose convergence rate is dictated by the connectivity level, the transient response of the proposed scheme is independent of the underlying topology.

Remark 4: In case of M-dimensional agent states, that is $x_i \in \Re^M$, $u_i \in \Re^M$ and $f_i : \Re^M \to \Re^M$, $g_i : \Re^M \to \Re^{M\times M}$, $d_i : \Re_+ \to \Re^M$, $i = 1, \ldots, N$ and $x_0 : \Re_+ \to \Re^M$, the formation control problem with prescribed performance can be solved following a similar design procedure, under a controllability assumption that the matrices $g_i(\cdot)$, $i = 1, \ldots, N$ are positive (or negative) definite. More specifically, we define the neighborhood error feedback:

$$e_i = \sum_{j \in N_i} a_{ij}\left(x_i - x_j + c_{ij}\right) + b_i\left(x_i - x_0 + c_{i0}\right) \in \Re^M$$

for $i = 1, \ldots, N$, from which, employing the Kronecker product $\otimes$, we obtain the overall neighborhood error vector:

$$\bar{e} = \left[e_1^T, \ldots, e_N^T\right]^T := \left(\left(L + B\right) \otimes I_M\right)\left(\bar{x} - \bar{x}_0 + \bar{c}\right),$$

where $\bar{x} = \left[x_1^T, \ldots, x_N^T\right] \in \Re^{NM}$ is the overall state vector of the multi-agent system, $\bar{x}_0 = \left[x_0^T, \ldots, x_0^T\right]^T \in \Re^{NM}$ and $\bar{c}$ denotes the relative offsets with respect to the leader, as dictated by the desired formation. Adopting element-wise for each neighborhood error $e_i := \left[e_{i1}, \ldots, e_{iM}\right]^T$ the corresponding performance functions $\rho_{ij}(t) = \left(\rho_{ij0} - \rho_\infty\right) e^{-lt} + \rho_\infty$, $i = 1, \ldots, N$ and $j = 1, \ldots, M$ such that $\left|e_{ij}(0)\right| < \rho_{ij0}$, $i = 1, \ldots, N$ and $j = 1, \ldots, M$ with l, ρ_∞ incorporating the desired transient and steady state specifications as presented previously, it can be easily verified that the distributed control scheme:

$$u_i\left(e_i, t\right) = -k_i \left[\ln\left(\frac{1 + \frac{e_{i1}}{\rho_{i1}(t)}}{1 - \frac{e_{i1}}{\rho_{i1}(t)}}\right), \ldots, \ln\left(\frac{1 + \frac{e_{iM}}{\rho_{iM}(t)}}{1 - \frac{e_{iM}}{\rho_{iM}(t)}}\right)\right]^T$$

with $k_i > 0$, $i = 1, \ldots, N$, guarantees $\left|e_{ij}(t)\right| < \rho_{ij}(t)$, $\forall t \geq 0$, $i = 1, \ldots, N$ and $j = 1, \ldots, M$ as well as the boundedness of all closed loop signals.

IV. Simulation Results

To demonstrate the proposed distributed formation control strategy, we consider a group of $N = 5$ planar 2 DOF agents in the cartesian workspace (i.e., $p_i = \left[x_i, y_i\right]^T \in \Re^2$, $i = 1, \ldots, 5$) with the following nonlinear model:

$$\dot{p}_i = f\left(p_i\right) + g\left(p_i\right) u_i + d\left(t\right),\ i = 1, \ldots, 5$$

where $f\left(p_i\right) = \left[x_i y_i + e^{-x_i^2 - y_i^2}, x_i^2 - y_i^2 + \sin\left(x_i y_i\right)\right]^T$, $g\left(p_i\right) = \mathrm{diag}\left(\left[1 + x_i^2, 1 + y_i^2\right]\right)$ and $d(t) = \left[\sin\left(3t\right), \cos\left(4t\right)\right]^T$ represents external disturbances. Additionally, a more realistic case is considered with the state measurements been corrupted by Gaussian noise with standard deviation 5% of the actual signals. Furthermore, the command/reference trajectory (leader node) is a smooth representation $x_0(t) := \left[x_d(t), y_d(t)\right]^T \in \Re^2$ of the acronym of our laboratory **CSL** (i.e., Control Systems Laboratory) with constant linear velocity (i.e., $\sqrt{\dot{x}_d^2(t) + \dot{y}_d^2(t)} = \text{const}$). Moreover, the desired formation corresponds to a regular pentagon with the leader at its center (see Fig. 1). Finally, to strengthen our argument that the performance of the proposed control scheme is independent of the underlying topology, as long as the connectivity assumption holds, we considered three different connected communication graphs as depicted in Fig. 1.

More specifically, we require steady state errors of no more than 0.1 m and minimum speed of convergence as obtained by the exponential e^{-2t}. Thus, following Remark 4 for multi dimensional agent states, we selected the performance functions $\rho_i^j(t) = \left(\rho_{i0}^j - \rho_\infty\right) e^{-lt} + \rho_\infty$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$ with $l = 2$, $\rho_\infty = 0.4 \sin^2\left(\frac{\pi}{22}\right) \approx 0.0081$ (as obtained by (6)) and $\rho_{i0}^j > \left|e_i^j(0)\right|$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$. Finally, we chose $k_i^j = 4.0$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$ to yield reasonable control effort. The simulation results are illustrated in Figs. 2-4 where the evolution of the neighborhood error metrics $e_i^j(t)$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$ is depicted along with the imposed performance bounds by the selected performance functions $\rho_i^j(t)$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$. As it was predicted by the theoretical analysis, the formation control problem with prescribed transient and steady state performance is solved irrespectively of the graph topology and despite the presence of external disturbances as well as the lack of knowledge of the agents' dynamics. Finally, a short video of the aforementioned simulation results as well as of a more aggressive case (i.e., a faster desired trajectory and a quicker transient response) can be found in: `http://youtu.be/0xC8vSDL_eg`.

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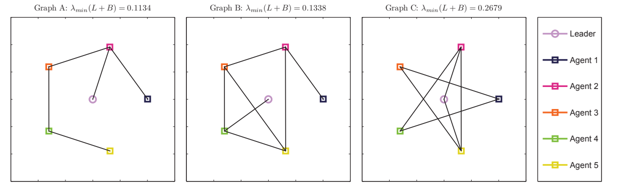

Fig. 1: The desired formation and the three considered communication graphs with different $\lambda_{\min}(L+B)$.

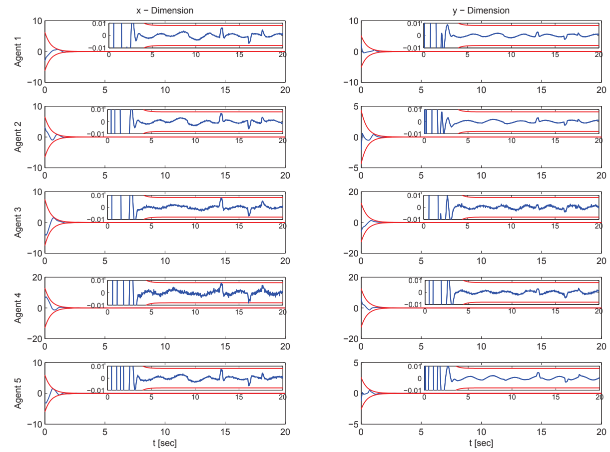

Fig. 2: Graph A: The evolution of the error metrics $e_i^j(t)$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$ (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots.

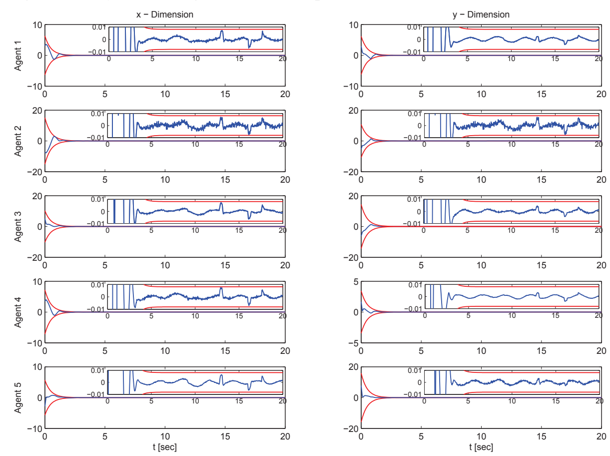

Fig. 3: Graph B: The evolution of the error metrics $e_i^j(t)$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$ (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots.

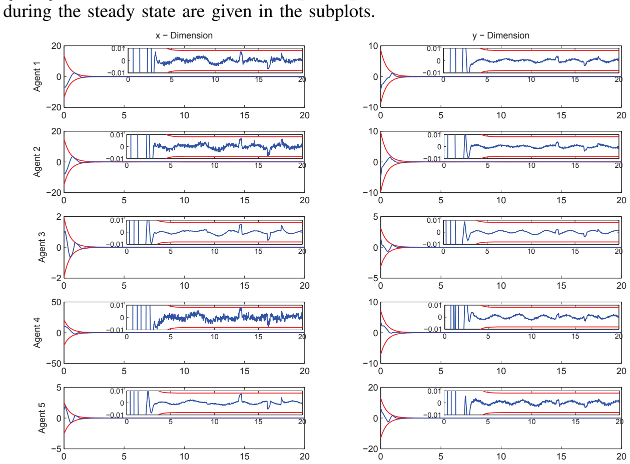

Fig. 4: Graph C: The evolution of the error metrics $e_i^j(t)$, $i = 1, \ldots, 5$ and $j \in \{x, y\}$ (blue lines) along with the imposed performance bounds (red lines). Details during the steady state are given in the subplots.