Robust Model-Free Formation Control with Prescribed Performance and Connectivity Maintenance for Nonlinear Multi-Agent Systems

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***Abstract*— In this paper, we consider the formation control problem for a generic class of first-order nonlinear multi-agent systems, under an undirected communication model and connectivity constraints. More specifically, we design a decentralized model-free control protocol in the sense that each agent utilizes only local relative state information from its neighborhood set to calculate its own control signal, without incorporating any prior knowledge of the model nonlinearities/disturbances or any approximation structures to acquire such knowledge. Assuming that initially the graph is strongly connected, the proposed scheme guarantees that all initial communication links are maintained, that is all pairs of agents, initially forming an edge in the graph, remain within a distance representing the communication capabilities of the agents, preserving thus the strong connectiveness for all time. Additionally, the transient and steady state response is solely determined by certain designer-specified performance functions and is fully decoupled by the agents' dynamic model, the underlying graph topology and the control gains selection, which further relaxes the control design procedure. Finally, the proposed methodology results in a low complexity design. Actually, it is a static scheme involving very few and simple calculations to output the control signal, thus making its distributed implementation straightforward.**

I. INTRODUCTION

Multi-agent systems have emerged as an inexpensive and robust way of addressing a wide variety of tasks, ranging from exploration, surveillance, and reconnaissance to cooperative construction and manipulation. The success of these systems relies on efficient information exchange and coordination between the members of the team. More specifically, their intriguing feature consists on the fact that each agent makes decisions solely on the basis of its local perception of the environment. Thus, a challenging task is to design the decentralized control approach for certain global goals in the presence of limited information exchanges. In this direction, drawing some enlightenments from biological observations, distributed cooperative control of multi-agent systems has received considerable attention during the last two decades (see the seminal works [1]–[3] for example). In particular, the leader-follower scheme, according to which the following agents aim at reaching a consensus with the leader's state, employing only locally available information, has become very popular, since in the absence of any central control system and without global coordinate information, following a leader is an accountable motivation.

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Although the majority of the works on distributed cooperative control consider known and simple dynamic models, there exist, however, many practical engineering systems which cannot be modeled accurately and which are constantly subject to environmental disturbances. Thus, taking into account the inherent model uncertainties when designing robust distributed control schemes is of paramount importance. On the other hand, extending towards this direction the existing control schemes for ideal models, becomes a very challenging task on account of the increasing design complexity from the interacting system dynamics as reflected by the local intercourse specifications. Nevertheless, multi-agent consensus/synchronization/formation control for systems with unknown nonlinear dynamics and disturbances was studied in [4], [5] employing neuro/fuzzy approximating structures to compensate for the model uncertainties. Unfortunately, the aforementioned approaches inherently introduce certain issues, common in all approximation based control schemes, affecting closed loop stability and robustness. Another important issue associated with distributed cooperative control schemes for multi-agent systems under model uncertainties, concerns the transient and steady state response of the closed loop system. Traditionally, the synchronization/consensus error is proven to convergence within a residual set, whose size depends on control design parameters and some unknown (though bounded) terms induced by the model uncertainties. However, no systematic procedure exists to accurately compute the required upper bounds, thus making the a priori selection of the aforementioned control parameters to satisfy certain steady state behavior, practically impossible. Moreover, the transient behavior is difficult to establish analytically as it is affected heavily by the agents' model dynamics and the status of the overall underlying interaction topology, both of which are considered unknown. To the best of the authors' knowledge, the transient performance problem was first relaxed for single integrator multi-agent systems in [6], following the prescribed performance notion.

Although communication graphs have long served as models of local interactions in multi-agent systems, their connectivity property was initially imposed by assumption [1]–[3] rather than treated as an extra control objective. Such assumption may seem valid for static communication networks, however, it is not realistic in case of mobile networks, where communication between the agents depends on their distance. Motivated by the need to prevent the graph from partitioning, centralized control protocols employing the Laplacian matrix were initially developed in [3], [7] to ensure connectivity. However, global information of the underlying topology was required to compute the graph Laplacian. Subsequently, a bulk of recent works [8]–[11] considered the distributed connectivity maintenance problem. A common point in the aforementioned references is the use of potential fields that force agents that constitute a distance-based communication link, to remain within a certain distance for all time. More specifically, whenever agents tend to move away from the communication threshold distance the potential field provides a guarantee for edge maintenance and thus connectivity. Unfortunately though, all aforementioned approaches considered simple single integrator dynamics without incorporating any transient and steady state performance specifications as well. In the same spirit, extending those approaches towards uncertain and complex nonlinear dynamics becomes a very challenging task on account of the increasing design complexity of potential fields, induced by the interacting system dynamics.

In this work, we propose a distributed control protocol for uncertain nonlinear multi-agent systems that creates arbitrarily fast and maintains with arbitrary accuracy a desired feasible formation without violating the connectivity constraints. The developed scheme exhibits the following important characteristics. First, it is purely decentralized in the sense that the control signal of each agent is calculated based solely on local relative state information from its neighborhood set. Furthermore, its complexity proves to be considerably low. Very few and simple calculations are required to output the control signal. Additionally, it does not require any prior knowledge of the agent's dynamic model and no estimation models are employed to acquire such knowledge. Moreover, contrary to the related works on multi-agent systems, the transient and steady state response is fully decoupled by the underlying graph topology, the control gains selection and the agents' model uncertainties. In particular, the achieved performance and the connectivity maintenance are a priori and explicitly imposed by certain designer-specified performance functions, thus simplifying significantly the selection of the control gains. Tuning of the controller gains is only confined to achieving reasonable control effort.

II. Problem Statement and Preliminaries

We consider a multi-agent group comprised of a leader and N followers, obeying a first order nonlinear dynamic model, described as follows:

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i + d_i(t), \, i = 1, \ldots, N \qquad (1)$$

where $x_i \in \Re$, $i = 1, \ldots, N$ denotes the state of each agent, $f_i, g_i : \Re \to \Re$, $i = 1, \ldots, N$ are unknown locally Lipschitz vector fields, $u_i \in \Re$, $i = 1, \ldots, N$ is the control input of each agent and $d_i : \Re_+ \to \Re$, $i = 1, \ldots, N$ represent piecewise continuous and bounded external disturbances. A sufficient controllability condition is further assumed for the control gain functions $g_i(x_i)$, $i = 1, \ldots, N$ (i.e., $g_i(x_i) \geq g_i^\star > 0$, $\forall x_i \in \Re$, $i = 1, \ldots, N$ for some unknown positive constants $g_i^\star$, $i = 1, \ldots, N$).

An undirected graph $G = (V, E)$ is used to model the communication among the agents, where $V = \{v_1, \ldots, v_N\}$ denotes the set of vertices that represent each agent. The set of edges is denoted as $E \subseteq V \times V$ and the graph is assumed to be simple, i.e. $(v_i, v_i) \notin E$ (there exist no self loops). The adjacency matrix associated with the graph G is denoted as $A = [a_{ij}] \in \Re^{N\times N}$ with $a_{ij} \in \{0, 1\}$, $i, j = 1, \ldots, N$. If $a_{ij} = 1$ then the agent i obtains information regarding the state of the j-th agent (i.e., $(v_i, v_j) \in E$), whereas if $a_{ij} = 0$ then there is no state-information flow from agent j to agent i (i.e., $(v_i, v_j) \notin E$). Furthermore, the set of neighbors of a vertex v_i is denoted by $N_i = \{v_j : (v_i, v_j) \in E\}$ and the degree matrix is defined as $D = \mathsf{diag}([D_i]) \in \Re^{N\times N}$ with $D_i = \sum_{j\in N_i} a_{ij}$. Moreover, since the graph is undirected, the adjacency is a mutual relation, thus A is symmetric, i.e., $a_{ij} = a_{ji}$, $\forall i, j = 1, \ldots, N$. In this respect, the Laplacian matrix of the graph, which is denoted by $L = D - A \in \Re^{N\times N}$, is also symmetric. Additionally, the state of the leader node (labeled v_0) is given by $x_0 : \Re_+ \to \Re$ and is further assumed to be smooth and bounded. However, the desired command/reference trajectory information is only provided to a subgroup of the N agents. The access of the following agents to the leader's state is modeled by a diagonal matrix $B = \mathsf{diag}([b_1, b_2 \ldots, b_n]) \in \Re^{N\times N}$. If b_i, $i \in \{1, 2, \ldots, N\}$ is equal to 1, then the i-th agent obtains state-information from the leader node; otherwise if b_i, $i \in \{1, 2, \ldots, N\}$ equals to 0, then the i-th agent cannot obtain state-information from the leader node. In this way, we may also define the augmented graph as $\bar{G} = (\bar{V}, \bar{E})$, where $\bar{V} = V \cup \{v_0\}$ and $\bar{E} = E \cup \{(v_i, v_0) : b_i = 1\} \subseteq \bar{V} \times \bar{V}$ as well as the augmented set of neighbors $\bar{N}_i = \{v_j : (v_i, v_j) \in \bar{E}\}$, $i = 1, \ldots, N$. Moreover, denoting by P the total number of distinct edges in $\bar{E}$ (i.e., $\bar{E} = \{\epsilon_1, \ldots, \epsilon_P\}$) and assuming any direction on each edge (i.e., each distinct edge is given a tail and a head), we may define the $N \times P$ incidence matrix $S_G = [s_{ij}]$, where:

$$s_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is the head of } \epsilon_j \\ -1, & \text{if } v_i \text{ is the tail of } \epsilon_j \\ 0, & \text{otherwise} \end{cases} .$$

Since the communication capabilities of the agents are inherently limited, every agent is aware of the state of only those agents in its augmented neighboring set $\bar{N}_i$ that also lie within its sensing radius $\bar{l}$. In this respect, assuming that initially the augmented graph is strongly connected, the control protocol should also guarantee that all initial edges are maintained, in the sense that all pairs of agents, initially forming an edge, remain within distance $\bar{l}$, preserving thus the strong connectiveness of the augmented graph for all $t \geq 0$. Hence, we formulate the robust formation control problem with prescribed performance under the aforementioned connectivity constraints as follows: *design a distributed control protocol for (1), considering model uncertainties and limited communication capabilities, such that a constant formation, described by the desired relative offsets* c_{ij}, $\forall (v_i, v_j) \in \bar{E}$, *and compatible with the agents' communication capabilities (in the sense of [10], i.e.,* $|c_{ij}| < \bar{l}$, $\forall (v_i, v_j) \in \bar{E}$*),* *is established arbitrarily fast and maintained with arbitrary accuracy while keeping the distance between any pair* $(v_i, v_j) \in \bar{E}$ *less that* $\bar{l}$. Finally, to solve the aforementioned multi-agent control problem, the following assumption is required on the graph topology.

Assumption: The augmented communication graph $\bar{G}$ is initially strongly connected and $|x_i(0) - x_j(0)| < \bar{l}$, $\forall (v_i, v_j) \in \bar{E}$.

Remark 1: Proximity graphs of connectivity, where edges are established between agents that lie within distance less than a radius representing the communication capabilities of the agents, are of particular importance in mobile multi-agent systems. It should be noticed that such graphs are dynamic since edges may appear or disappear as the agents' state evolves. However, in this work we restrict the set of edges to be static (i.e., we do not consider adding edges) and we try to guarantee that all initially connected agents (according to the aforementioned assumption) remain within distance $\bar{l}$ for all $t > 0$ (i.e., all initial edges are preserved), securing thus the connectivity property that the graph initially satisfies.

A. Dynamical Systems

Consider the initial value problem:

$$\dot{\psi} = H(t, \psi), \ \psi(0) = \psi^0 \in \Omega_\psi \tag{2}$$

with $H : \Re_+ \times \Omega_\psi \to \Re^n$ where $\Omega_\psi \subset \Re^n$ is a non-empty open set.

Definition 1: [12] A solution $\psi(t)$ of the initial value problem (2) is maximal if it has no proper right extension that is also a solution of (2).

As an example, consider the initial value problem $\dot{\psi} = \psi^2$, $\psi(0) = 1$, whose solution is $\psi(t) = \frac{1}{1-t}$, $\forall t \in [0, 1)$. The solution is maximal since it cannot be defined for $t > 1$. Stated otherwise, there is no proper extension of $\psi(t)$ to the right of $t = 1$ that is also a solution of the original initial value problem.

Theorem 1: [12] Consider the initial value problem (2). Assume that $H(t, \psi)$ is: a) locally Lipschitz on ψ for almost all $t \in \Re_+$, b) piecewise continuous on t for each fixed $\psi \in \Omega_\psi$ and c) locally integrable on t for each fixed $\psi \in \Omega_\psi$. Then, there exists a maximal solution $\psi(t)$ of (2) on the time interval $[0, \ t_{\max})$ with $t_{\max} > 0$ such that $\psi(t) \in \Omega_\psi$, $\forall t \in [0, \ t_{\max})$.

Proposition 1: [12] Assume that the hypotheses of Theorem 1 hold. For a maximal solution $\psi(t)$ on the time interval $[0, \ t_{\max})$ with $t_{\max} < \infty$ and for any compact set $\Omega'_\psi \subset \Omega_\psi$ there exists a time instant $t' \in [0, \ t_{\max})$ such that $\psi(t') \notin \Omega'_\psi$.

III. Control Scheme

Let us define the stack vector $\bar{e} = [e_k] \in \Re^P$ of the relative neighborhood errors $e_k = x_i - x_j + c_{ij}$, $k = 1, \ldots, P$ for all distinct edges $(v_i, v_j) \in \bar{E}$ of connected agents. In this work, prescribed performance control will be adopted in order to: i) achieve predefined transient and steady state response for each relative neighborhood error e_k as well as ii) preserve initially connected agents within distance $\bar{l}$. Following [13], prescribed performance is achieved when the relative neighborhood errors e_k, $k = 1, \ldots, P$ evolve strictly within a predefined region that is bounded by absolutely decaying functions of time, called performance functions. The mathematical expression of prescribed performance is given by the following inequalities:

$$-\underline{M}_k \rho_k(t) < e_k(t) < \overline{M}_k \rho_k(t), \ \forall t \geq 0 \tag{3}$$

for all $k = 1, \ldots, P$, where:

$$\rho_k(t) = (1 - \tfrac{\rho_\infty}{\max\{\underline{M}_k, \overline{M}_k\}})e^{-lt} + \tfrac{\rho_\infty}{\max\{\underline{M}_k, \overline{M}_k\}} \tag{4}$$

are designer-specified, smooth, bounded and decreasing functions of time with l, ρ_∞ positive parameters incorporating the desired transient and steady state performance specifications respectively, and $\underline{M}_k$, $\overline{M}_k$ are positive parameters selected appropriately to satisfy the local connectivity constraints, as presented in the sequel. In particular, the decreasing rate of $\rho_k(t)$, which is affected by the constant l, introduces a lower bound on the speed of convergence of $e_k(t)$, $k = 1, \ldots, P$. Furthermore, the constant ρ_∞ can be set arbitrarily small, thus achieving practical convergence of $e_k(t)$, $k = 1, \ldots, P$ to zero. Additionally, under the initial connectivity assumption that $|x_i(0) - x_j(0)| < \bar{l}$, $\forall (v_i, v_j) \in \bar{E}$, we select the parameters $\underline{M}_k$, $\overline{M}_k$, $k = 1, \ldots, P$ as:

$$\underline{M}_k = \bar{l} - c_{ij} \text{ and } \overline{M}_k = \bar{l} + c_{ij}. \tag{5}$$

Apparently, since the desired formation is compatible with the connectivity constraints (i.e., $|c_{ij}| < \bar{l}$, $\forall (v_i, v_j) \in \bar{E}$), the aforementioned selection ensures that $\underline{M}_k, \overline{M}_k > 0$, $k = 1, \ldots, P$ and consequently that:

$$-\underline{M}_k \rho_k(0) < e_k(0) < \overline{M}_k \rho_k(0), \ k = 1, \ldots, P. \tag{6}$$

Moreover, guaranteeing (3) and employing the decreasing property of $\rho_k(t)$, we also obtain:

$$-\underline{M}_k < e_k(t) < \overline{M}_k, \ k = 1, \ldots, P$$

which further leads via (5) to the satisfaction of the connectivity constraints (i.e., $-\bar{l} < x_i(t) - x_j(t) < \bar{l}, \forall (v_i, v_j) \in \bar{E}$ for all $t \geq 0$).

Before we proceed with the main results of this work, we shall define the normalized relative neighborhood errors, which are necessary in the subsequent analysis, as follows:

$$\xi(\bar{e}, t) = \begin{bmatrix} \xi_1(e_1, t) \\ \vdots \\ \xi_P(e_P, t) \end{bmatrix} := \begin{bmatrix} \frac{e_1}{\rho_1(t)} \\ \vdots \\ \frac{e_P}{\rho_P(t)} \end{bmatrix} \triangleq (\rho(t))^{-1} \bar{e} \tag{7}$$

where $\rho(t) = \mathsf{diag}\left([\rho_k(t)]_{k=1,\ldots,P}\right)$. In the sequel, we propose a distributed control protocol, without incorporating any information on the model nonlinearities or the external disturbances, that guarantees $-\underline{M}_k \rho_k(t) < e_k(t) < \overline{M}_k \rho_k(t)$, $k = 1, \ldots, P$ for all $t \geq 0$ and leads consequently to the solution of the robust formation control problem with prescribed performance under local connectivity constraints for the considered multi-agent system.

Theorem 2: Given the relative neighborhood errors $e_k = x_i - x_j + c_{ij}$, $k = 1, \ldots, P$ for all distinct edges $(v_i, v_j) \in \bar{E}$ of initially connected agents, as well as the appropriately selected corresponding functions $\rho_k(t)$ and positive parameters $\underline{M}_k, \overline{M}_k > 0$, $k = 1, \ldots, P$, the distributed control protocol:

$$u(\bar{e}, t) = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} = -\mu S_G (\rho(t))^{-1} r(\xi)\varepsilon(\xi), \ \mu > 0 \tag{8}$$

where:

$$r(\xi) = \text{diag}\left(\left[\frac{\frac{1}{\underline{M}_k} + \frac{1}{\overline{M}_k}}{\left(1 + \frac{\xi_k}{\underline{M}_k}\right)\left(1 - \frac{\xi_k}{\overline{M}_k}\right)}\right]_{k=1,\ldots,P}\right) \tag{9}$$

$$\varepsilon(\xi) = \left[\ln\left(\frac{1 + \frac{\xi_1}{\underline{M}_1}}{1 - \frac{\xi_1}{\overline{M}_1}}\right), \ldots, \ln\left(\frac{1 + \frac{\xi_P}{\underline{M}_P}}{1 - \frac{\xi_P}{\overline{M}_P}}\right)\right]^T, \tag{10}$$

guarantees:

$$-\underline{M}_k \rho_k(t) < e_k(t) < \overline{M}_k \rho_k(t), \ k = 1, \ldots, P$$

for all $t \geq 0$, as well as the boundedness of all closed loop signals.

Proof: Let us initially define the open set:

$$\Omega_\xi = \left(-\underline{M}_1, \overline{M}_1\right) \times \cdots \times \left(-\underline{M}_P, \overline{M}_P\right).$$

In what follows, we proceed in two phases. First, the existence of a unique maximal solution $\xi(t)$ over the set Ω_ξ for a time interval $[0, \tau_{\max})$ (i.e., $\xi(t) \in \Omega_\xi$, $\forall t \in [0, \tau_{\max})$) is ensured. Then, we prove that the proposed control scheme (8)-(10) guarantees, for all $t \in [0, \tau_{\max})$: a) the boundedness of all closed loop signals as well as that b) $\xi(t)$ remains strictly within a compact subset of Ω_ξ, which leads by contradiction to $\tau_{\max} = \infty$ and consequently to the completion of the proof.

Differentiating (7) with respect to time, we obtain:

$$\dot{\xi} = (\rho(t))^{-1}\left(\dot{\bar{e}} - \dot{\rho}(t)\xi\right). \tag{11}$$

Furthermore, it can be easily verified that:

$$\bar{e} = S_G^T \bar{\delta}, \tag{12}$$

where $\bar{\delta} = \bar{x} - \bar{x}_0 + \bar{c}$ is the disagreement formation variable with $\bar{c}$ denoting the vector of relative offsets with respect to the leader. Thus, differentiating (12) with respect to time and substituting the dynamics of the agents (1) as well as the control input (8), we get:

$$\begin{aligned} \dot{\bar{e}} = \ & S_G^T \Big(f(\bar{x}) - \mu g(\bar{x}) S_G (\rho(t))^{-1} r(\xi)\varepsilon(\xi) \\ & + d(t) - \dot{\bar{x}}_0(t) \Big), \end{aligned} \tag{13}$$

where $f(\bar{x}) = [f_1(x_1), \ldots, f_N(x_N)]^T$, $g(\bar{x}) = \text{diag}([g_1(x_1), \ldots, g_N(x_N)])$, $d(t) = [d_1(t), \ldots, d_N(t)]^T$ and $\bar{x}_0(t) = [x_0(t), \ldots, x_0(t)]^T \in \Re^N$. Moreover, notice from (7), (12) as well as the fact that $S_G S_G^T = L + B$, where L is the Laplacian matrix and B is the diagonal matrix denoting the connection of the following agents to the leader (it should also be noted that selecting the parameters $\underline{M}_k$, $\overline{M}_k$, $k = 1, \ldots, P$ according to (5) and owing to the assumption on the initial strong connectiveness, we guarantee that $L + B$ is positive definite for all $\xi \in \Omega_\xi$), we obtain:

$$\bar{x}(\xi, t) := \bar{x}_0(t) - \bar{c} + (L + B)^{-1} S_G \rho(t)\xi. \tag{14}$$

Thus, substituting (13) and (14) in (11), the closed loop dynamical system of ξ may be written as:

$$\begin{aligned} \dot{\xi} &= h(t, \xi) \\ &= (\rho(t))^{-1}\Big(S_G^T\big(f(\bar{x}) + d(t) - \dot{\bar{x}}_0(t) \\ &\quad - \mu g(\bar{x}) S_G (\rho(t))^{-1} r(\xi)\varepsilon(\xi)\big) - \dot{\rho}(t)\xi\Big). \end{aligned} \tag{15}$$

Phase A. Selecting the parameters $\underline{M}_k$, $\overline{M}_k$, $k = 1, \ldots, P$ according to (5), we guarantee that the set Ω_ξ is nonempty and open and moreover that $\xi(0) \in \Omega_\xi$ as shown in (6). Additionally, h is continuous on t and locally Lipschitz on ξ over the set Ω_ξ. Therefore, the hypotheses of Theorem 1 stated in Subsection II-A hold and the existence of a maximal solution $\xi(t)$ of (15) on a time interval $[0, \tau_{\max})$ such that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, \tau_{\max})$ is ensured.

Phase B. We have proven in Phase A that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, \tau_{\max})$ and more specifically that:

$$\xi_k(t) = \frac{e_k(t)}{\rho_k(t)} \in \left(-\underline{M}_k, \overline{M}_k\right), \ k = 1, \ldots, P \tag{16}$$

for all $t \in [0, \tau_{\max})$, from which we obtain that $e_k(t)$ is absolutely bounded by $\max\left\{\underline{M}_k, \overline{M}_k\right\}\rho_k(t)$, $k = 1, \ldots, P$ for all $t \in [0, \tau_{\max})$. Similarly, we may prove from (14), that

$$\bar{x}(\xi, t) \in \Omega_x, \forall t \in [0, \tau_{\max}) \tag{17}$$

with:

$$\Omega_x = \left\{x \in \Re^N : \|x\| \leq \sup_{t \geq 0}\{\|\bar{x}_0 - \bar{c}\|\} + \frac{\sqrt{N}\max\{\underline{M}_k, \overline{M}_k\}}{\lambda_{\min}((L+B))}\right\} \tag{18}$$

denoting a compact set with size independent of $\tau_{\max}$, owing to the boundedness of $\bar{x}_0(t)$. Furthermore, owing to (16), the error vector (10) is well defined for all $t \in [0, \tau_{\max})$. Hence, let us define the following hybrid quadratic-integral Lyapunov function:

$$V_\varepsilon = \frac{\varepsilon^T\varepsilon}{2} + \eta\int_0^1 \theta\bar{\delta}^T\rho(t)\left(g\left(\bar{x}_0 - \bar{c} + \theta(L+B)^{-1}\bar{\delta}\right)\right)^{-1}\bar{\delta}d\theta.$$

with η denoting a positive constant to be defined in the sequel. Similarly to [14], differentiating V_ε with respect to time as well as substituting (9), (15) and employing $\bar{\delta}^T S_G = \bar{e}^T$, we arrive at:

$$\begin{aligned} \dot{V}_\varepsilon &= -\mu\varepsilon^T(\xi) r(\xi)(\rho(t))^{-1} S_G^T g(\bar{x}) S_G (\rho(t))^{-1} r(\xi)\varepsilon(\xi) \\ &\quad + \varepsilon^T(\xi) r(\xi)(\rho(t))^{-1}\Big(S_G^T\big(f(\bar{x}) + d(t) - \dot{\bar{x}}_0\big) - \dot{\rho}(t)\xi\Big) \\ &\quad - \mu\eta\bar{e}^T(\rho(t))^{-1} r(\xi)\varepsilon(\xi) + \eta\bar{\delta}^T(g(\bar{x}))^{-1} f(\bar{x}) \\ &\quad + \eta\int_0^1 \dot{\bar{x}}_0^T\left(g\left(\bar{x}_0 - \bar{c} + \theta(L+B)^{-1}\bar{\delta}\right)\right)^{-1}\bar{\delta}d\theta. \end{aligned}$$

Exploiting: i) the Extreme Value Theorem, owing to (17) and the fact that the vector fields $f(\cdot)$, $(g(\cdot))^{-1}$ are continuous and Ω_x is compact, as well as ii) the boundedness of $d(t)$, $\dot{\bar{x}}_0(t)$, $(\rho(t))^{-1}$ and $\dot{\rho}(t)$, there exists a positive constant F_ε independent of $\tau_{\max}$, such that:

$$\left\|(\rho(t))^{-1}\left(S_G^T\left(f(\bar{x}) + d(t) - \dot{\bar{x}}_0(t)\right) - \dot{\rho}(t)\xi\right)\right\| \leq F_\varepsilon$$

for all $(\bar{x}, t) \in \Omega_x \times \Re_+$. Moreover, utilizing $\bar{\delta} = (L+B)^{-1} S_G \bar{e}$ and $\bar{e} = \rho(t)\xi$ from (7) and (12) respectively, we obtain:

$$\left\| \eta \bar{\delta}^T (g(\bar{x}))^{-1} f(\bar{x}) + \eta \int_0^1 \dot{\bar{x}}_0^T(t) \left(g\left(\bar{x}_0(t) - \bar{c} + \theta (L+B)^{-1} \bar{\delta}\right)\right)^{-1} \bar{\delta} d\theta \right\| \leq \eta \|\xi\| F_\delta$$

with F_δ a positive constant satisfying:

$$\left\| \rho(t) S_G^T (L+B)^{-1} (g(\bar{x}))^{-1} f(\bar{x}) \right\| + \lambda_{\max}\left((g(\bar{x}))^{-1}\right) \left\| \rho(t) S_G^T (L+B)^{-1} \dot{\bar{x}}_0(t) \right\| \leq F_\delta$$

for all $(\bar{x}, t) \in \Omega_x \times \Re_+$. Notice further that the matrix $r(\xi)(\rho(t))^{-1} S_G^T g(\bar{x}) S_G (\rho(t))^{-1} r(\xi)$ is positive semidefinite by construction [8]. Hence, $\dot{V}_\varepsilon$ becomes:

$$\begin{aligned} \dot{V}_\varepsilon \leq & -\frac{1}{2}\mu\eta\xi^T r(\xi)\varepsilon(\xi) + \|r(\xi)\varepsilon(\xi)\| F_\varepsilon \\ & -\frac{1}{2}\mu\eta\xi^T r(\xi)\varepsilon(\xi) + \eta\|\xi\| F_\delta. \end{aligned}$$

Therefore, selecting $\eta > \frac{4F_\varepsilon}{\mu \min\{\underline{M}_k, \overline{M}_k\}}$ and employing the fact that $\xi^T r(\xi)\varepsilon(\xi) > 0$, $\forall \xi \neq 0$ and $r(\xi)$ is diagonal with $\lambda_{\min}(r(\xi)) = \min\left\{\frac{1}{\underline{M}_k} + \frac{1}{\overline{M}_k}\right\}$ by construction, we conclude that $\dot{V}_\varepsilon < 0$ whenever $\|\varepsilon(\xi)\| > \max\left\{\ln(3), \frac{F_\delta}{\mu}\right\}$. Thus, owing to the positive definiteness and radially unboundedness of V_ε with respect to ε, we obtain:

$$\|\varepsilon(t)\| \leq \bar{\varepsilon} = \max\left\{\|\varepsilon(0)\|, \max\left\{\ln(3), \frac{F_\delta}{\mu}\right\}\right\} \quad (19)$$

for all $t \in [0, \tau_{\max})$, from which, by taking the inverse logarithmic function in (10), we get:

$$-\underline{M}_k < \frac{e^{-\bar{\varepsilon}}-1}{e^{-\bar{\varepsilon}}+1}\underline{M}_k = \underline{\xi}_k \leq \xi_k(t) \leq \bar{\xi}_k = \frac{e^{\bar{\varepsilon}}-1}{e^{\bar{\varepsilon}}+1}\overline{M}_k < \overline{M}_k \quad (20)$$

with $k = 1, \ldots, P$ for all $t \in [0, \tau_{\max})$. Moreover, it can be easily verified that the control signal (8) remains bounded for all $t \in [0, \tau_{\max})$.

Up to this point, what remains to be shown is that $\tau_{\max}$ can be extended to ∞. In this direction, notice by (20) that $\xi(t) \in \Omega_\xi^{'}$, $\forall t \in [0, \tau_{\max})$, where the set $\Omega_\xi^{'} = \left[\underline{\xi}_1, \bar{\xi}_1\right] \times \cdots \times \left[\underline{\xi}_P, \bar{\xi}_P\right]$: is a nonempty and compact subset of Ω_ξ. Hence, assuming $\tau_{\max} < \infty$ and since $\Omega_\xi^{'} \subset \Omega_\xi$, Proposition 1 in Subsection II-A dictates the existence of a time instant $t^{'} \in [0, \tau_{\max})$ such that $\xi\left(t^{'}\right) \notin \Omega_\xi^{'}$, which is a clear contradiction. Therefore, $\tau_{\max} = \infty$. Thus, all closed loop signals remain bounded and moreover $\xi(t) \in \Omega_\xi^{'} \subset \Omega_\xi$, $\forall t \geq 0$. Finally, multiplying (20) by $\rho_k(t)$, we also conclude:

$$-\underline{M}_k\rho_k(t) < \underline{\xi}_k\rho_k(t) \leq e_k(t) \leq \bar{\xi}_k\rho_k(t) < \overline{M}_k\rho_k(t) \quad (21)$$

with $k = 1, \ldots, P$ for all $t \geq 0$ and consequently the solution of the robust formation control problem with prescribed performance under connectivity constraints for the considered multi-agent system. ■

Remark 2: From the aforementioned proof it can be deduced that the proposed control scheme achieves its goals without resorting to the need of rendering $\bar{\varepsilon}$ arbitrarily small (see (19)), by adopting extreme values of the control gain μ. More specifically, notice that (20) and consequently (21), which encapsulates the prescribed performance notion, hold no matter how large the finite bound $\bar{\varepsilon}$ is. In the same spirit, large model uncertainties can be compensated, as they affect only the size of $\bar{\varepsilon}$ through $\bar{F}_\delta$, but leave unaltered the achieved stability properties. Hence, the actual performance, which is solely determined by the performance functions $\rho_k(t)$, $k = 1, \ldots, P$ becomes isolated against model uncertainties, thus extending greatly the robustness of the proposed control scheme. Furthermore, unlike what is the common practice in the related literature (i.e., the control gains are tuned towards satisfying a desired performance, nonetheless without any a priori guarantees), the selection of the control gain μ is significantly simplified to adopting those values that lead to reasonable control effort. Finally, it should be noticed that contrary to the standard distributed control schemes for static graphs, whose convergence rate is dictated by the connectivity level, the transient response of the proposed scheme is independent of the underlying topology.

Remark 3: In case of M-dimensional agent states, the formation control problem with prescribed performance under connectivity constraints can be solved following a similar design procedure. More specifically, defining element-wise for each dimension the stack vector $\bar{e}^m = [e_k^m] \in \Re^P$ of the relative neighborhood errors $e_k^m = x_i^m - x_j^m + c_{ij}^m$, $k = 1, \ldots, P$ and $m = 1, \ldots, M$ for all distinct edges $(v_i, v_j) \in \bar{E}$ of connected agents, it can be easily verified that the following distributed control scheme $u^m(\bar{e}^m, t) = -\mu_m S_G (\rho^m(t))^{-1} r(\xi^m)\varepsilon(\xi^m)$ for $m = 1, \ldots, M$, with $\mu_m > 0$, where $\rho^m(t) = \mathsf{diag}\left([\rho_k^m(t)]_{k=1,\ldots,P}\right)$, $\xi^m = (\rho^m(t))^{-1}\bar{e}^m$, $r(\xi^m) = \mathsf{diag}\left(\left[\frac{\frac{1}{\underline{M}_k^m} + \frac{1}{\overline{M}_k^m}}{\left(1 + \frac{\xi_k^m}{\underline{M}_k^m}\right)\left(1 - \frac{\xi_k^m}{\overline{M}_k^m}\right)}\right]_{k=1,\ldots,P}\right)$, $\varepsilon(\xi^m) = \left[\ln\left(\frac{1 + \frac{\xi_1^m}{\underline{M}_1^m}}{1 - \frac{\xi_1^m}{\overline{M}_1^m}}\right), \ldots, \ln\left(\frac{1 + \frac{\xi_P^m}{\underline{M}_P^m}}{1 - \frac{\xi_P^m}{\overline{M}_P^m}}\right)\right]^T$ with the functions $\rho_k^m(t) = \left(1 - \frac{\rho_\infty}{\max\{\underline{M}_k^m, \overline{M}_k^m\}}\right)e^{-lt} + \frac{\rho_\infty}{\max\{\underline{M}_k^m, \overline{M}_k^m\}}$ and the parameters $\underline{M}_k^m, \overline{M}_k^m > 0$, $k = 1, \ldots, P$ and $m = 1, \ldots, M$ incorporating the desired performance specifications and the connectivity constraints respectively, guarantees:

$$-\underline{M}_k^m \rho_k^m(t) < e_k^m(t) < \overline{M}_k^m \rho_k^m(t), \forall t \geq 0$$

for $k = 1, \ldots, P$ and $m = 1, \ldots, M$, as well as the boundedness of all closed loop signals. Notice, however, in this case, where the prescribed performance specifications are imposed element-wise for each dimension, the distance $\bar{l}$ that defines the connectivity constraints is expressed in the L_∞ norm and not in the conventional Euclidean (i.e., L_2) norm (the same also holds for the assumption on the initial connectivity). Nonetheless, any M-dimensional spherical sensing area of radius ρ in the L_2 metric may be conservatively replaced by an M-dimensional ball set of radius $\bar{l} = \frac{\rho}{\sqrt{M}}$ in the L_∞ metric.

IV. Simulation Results

To demonstrate the efficiency of the proposed distributed control protocol, we consider a group of $N = 5$ planar 2 DOF agents in the cartesian workspace (i.e., $p_i = [x_i, y_i]^T \in \Re^2$, $i = 1, \ldots, 5$) with the following nonlinear model:

$$\dot{p}_i = f(p_i) + g(p_i)\, u_i + d(t),\ i = 1, \ldots, 5$$

where $f(p_i) = \left[x_i y_i + e^{-x_i^2 - y_i^2}, x_i^2 - y_i^2 + \sin(x_i y_i)\right]^T$, $g(p_i) = \mathrm{diag}\left(\left[1 + x_i^2, 1 + y_i^2\right]\right)$ and $d(t) = \begin{bmatrix} \sin(3t) \\ \cos(4t) \end{bmatrix}$. The command/reference trajectory (leader node) is a smooth representation $x_0(t) := [x_d(t), y_d(t)]^T \in \Re^2$ of the acronym of our laboratory **CSL** (i.e., Control Systems Laboratory) with constant linear velocity (i.e., $\sqrt{\dot{x}_d^2(t) + \dot{y}_d^2(t)} = \mathrm{const}$). Furthermore, all agents initialize from the state of the leading agent and the desired formation corresponds to a rectangular. Moreover, the desired distance between two neighboring agents is set equal to 0.9 and the connectivity constraints are represented by a square of edge 2 (i.e., $\bar{l} = 1$) with the corresponding agent at its center. Notice that the aforementioned formation problem under the considered communication constraints is very challenging as the desired formation is close to violating the connectivity constraints. We also require steady state errors of no more than 0.01 m and minimum speed of convergence as obtained by the exponential e^{-12t}. Thus, following (4), (5) and Remark 3 for multi-dimensional agent states, we appropriately selected the parameters $\underline{M}_k^m, \overline{M}_k^m$ as well as the functions $\rho_k^m(t)$, $k = 1, \ldots, 5$ and $m = 1, \ldots, 2$ with $l = 12$ and $\rho_\infty = 0.01$, to achieve the desired performance specifications and satisfy the communication constraints. Furthermore, we chose $\mu_1 = \mu_2 = 0.075$ to yield reasonable control effort. Finally, the evolution of the state of the agents along with the connectivity constraints are illustrated in Fig. 1. As it was predicted by the theoretical analysis, the formation control problem with prescribed performance under connectivity constraints is solved with bounded closed loop signals, despite the presence of external disturbances as well as the lack of knowledge of the agents' dynamics. Finally, a short video demonstrating the aforementioned simulation results can be found at: `http://youtu.be/OMhPlQiwI9k`

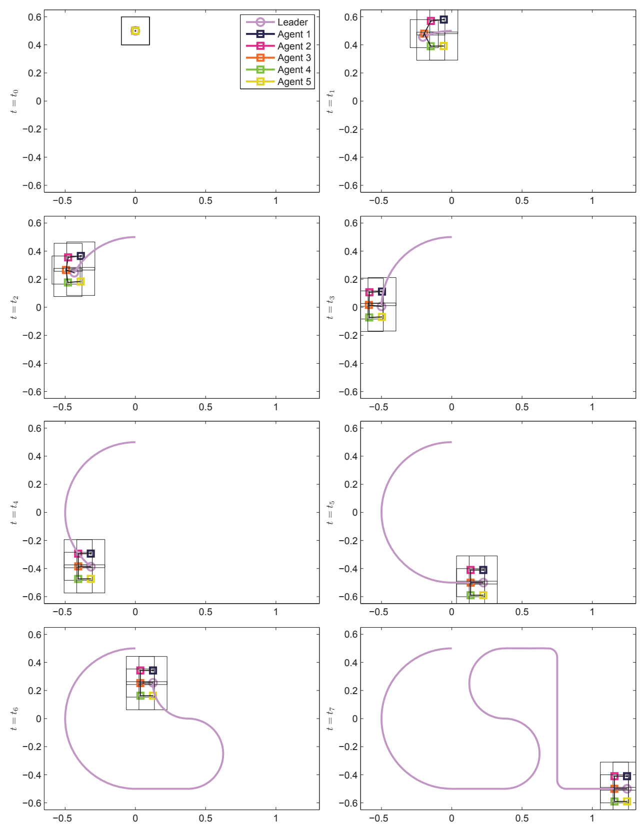

Fig. 1. The evolution of the multi agent system for 8 consecutive time instants.

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